AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 19, NUMBER 9 DECEMBER, 1951

The Fundamental Problems of Experimental Physics

JAMES HOUGH
University College, Hull, England
(Received February 27, 1951)

An elementary treatment of the nature of an experiment is given in order to demonstrate the various factors which must be considered in planning an experiment. Particular attention has been paid to the factors on which the accuracy of the final result depends; and it has been found desirable to introduce two new quantities, the reading and experimental limits, which give a measure of the maximum accuracy of a measurement and an experiment, respectively, obtainable with the particular apparatus.

INTRODUCTION

IN recent years it was decided to provide students with duplicated copies of instruction sheets for laboratory experiments in the Physics Department at University College, Hull, England. A supplement to these sheets was also issued, in which some of the more common problems of laboratory practice were discussed in detail, and to which the author contributed an introductory chapter on the more basic problems of experimental physics. It has been suggested that the material prepared for this purpose might be of interest to other teachers of physics, as it does not appear to be readily available in the standard textbooks of practical physics. In writing this article, the author is aware that it would be possible to extend any section to great length by a much more thorough discussion of the problems; but it appeared more useful to survey the whole field in an elementary manner in order to clarify the major factors involved in experimental physics.

Ever since the dawn of history, man has been concerned with the reason for his existence and with the surroundings in which he lives. In its most general form this is the central problem of philosophy; but, in physics, we are concerned with only one special branch of philosophy, *viz.*, the study of the material universe. The older name for physics, namely, Natural Philosophy, emphasizes this restriction. The study of the material universe must have started by early man noticing changes in his surroundings due to natural causes (e.g., the annual variation of the weather). Such observations, over a period of time, led to the discovery of a pattern in Nature and prepared the way for the development of the experimental method which consists in the study of the effects caused by known changes in the system. All real progress in science has come through the application of the experimental method; and, in physics, the stage has now been reached when nearly all experiments are quantitative. Consequently, the usual physical experiment involves the measurement of different quantities entering into the experiment and the derivation of a numerical quantity as the final result.

Students often blame poor apparatus for the low accuracy of their results, but it is usually the fault of their very poor experimental technique. An examination of the work of the great experimenters of the past, such as Cavendish and Joule, proves that amazingly accurate results can be obtained with very crude apparatus if it is used by a master. It is true that lack of time prevents a student making a really thorough investigation of each laboratory experiment, but it would be very valuable if time could be found in his course to enable him to carry out at least one experiment in such a way that the apparatus was made to yield its optimum accuracy.

As every experiment comprises measurements of different physical quantities, the present paper is divided into two parts, the first of which considers the problems involved in a single measurement and the second the way in which these are combined to form an experiment and the additional problems which an experiment presents.

PART I. A PHYSICAL MEASUREMENT

1. The Nature of a Measurement

To measure a quantity it is necessary to have defined a scale to represent the magnitude of the quantity and also some instrument with which to measure it; thus, to measure a length, a ruler is used as the instrument and the magnitude is expressed in centimeters. Although there are a vast number of instruments used for physical measurements, nearly all of them are based on pointer readings; i.e., they utilize the movement of a pointer over a scale and the observer is required only to judge the position of the pointer with respect to the scale. Examples are the measurement of length by judging the position of the end of an object with respect to a meter rule and the measurement of current by reading the position of an ammeter needle on the calibrated scale. Null methods are special cases of pointer readings in which balance is obtained when the pointer does not move when the experiment is performed; for example, a Wheatstone bridge is balanced if there is zero deflection of the galvanometer when the switch is closed.

It might well be asked why pointer readings are the most important type of physical measurements. The answer—as is fitting for an experimental science—is that experience has proved this type of measurement to be more reproducible than other types, particularly when the measurements are made by different observers; in other words, experience has shown that pointer readings are more consistent than other types of measurements. Pointer readings make few demands on the observer's judgment; and, to use philosophical language, they may be called objective measurements to distinguish them from subjective measurements in which the observer's judgment plays an important part. In elementary textbooks it is customary to illustrate the unreliability of subjective measurements by considering the use of the human hand as a thermometer and showing that it is more reliable as a guide to change of temperature than to the actual temperature. Unfortunately, in some fields of physics, it is not possible to avoid the use of subjective measurements; and it is certain that these are the most difficult fields in which to set up standards of measurement. The two principal fields of subjective measurement in physics are the color and brightness of light and the intensity and timbre of sound. In both these fields, the quantities to be measured are intimately connected with physiological sensation and so are essentially subjective. It is never good physics to use a subjective method unless the problem involves an essentially subjective quantity; and, even in these cases, there is a tendency to define an average estimate of a subjective quantity in terms of objective quantities and to make further measurements in terms of these. For example, in sound, an average ear has been defined to have specified threshold values for the minimum sound energy detectable at different frequencies. As these subjective branches of physics form a small part of physics only, it is not proposed to discuss their special problems in this general survey.

Bad lighting, an uncomfortable position, and fatigue of the observer all reduce the accuracy and reproducibility of pointer readings. Great care must be taken to see that pointer-reading errors are eliminated by systematic readings; for example, parallax errors in good quality electrical instruments are reduced by using the mirror incorporated in the scale to insure that

the line of observation is perpendicular to the scale.

2. Units

A pointer reading by itself is of no significance. For example, to say that the deflection of a tangent galvanometer is 55° tells us nothing about the current unless we have some further data on the instrument; if we know nothing about the instrument, we cannot even say that a current which produces a deflection of 55° is greater than one which produces a deflection of 20°. In order to be able to compare the currents, the law of the instrument must be known; and for a tangent galvanometer this law is $i=k\tan\theta$ in the familiar notation. If the actual magnitude of the current is required, the value of k must be known; and, in turn, this cannot be determined until the unit of current has been defined. In general, a pointer reading cannot be converted to a useful physical quantity until

(*i*) the law of the instrument is known. (Mathematically, if P is the quantity to be measured and s is the corresponding pointer reading, then $P=f(s)$ and the function $f(s)$ must be known);
(*ii*) the unit quantity has been defined.

The same arguments apply to null methods and the two equivalent conditions are

(*i*) the conditions for no deflection must be known;
(*ii*) one of the quantities compared must be known and this in turn implies that the unit quantity has been defined.

Although the definition of unit quantities is essentially arbitrary, it is the aim of any scientific work to build up a system of units that is simply interrelated and which involves the minimum number of numerical conversion factors. The original units adopted in scientific work were invented by the particular investigator to suit his particular problem; as a result of these pioneer investigations, knowledge was accumulated which led to a clearer picture of the problems involved and in the course of time to a desire for a systematic set of units common to all investigators. Naturally, owing to their commercial importance, units of mass and length were the first to be standardized; and so these units, together with that of time (easily standardized in terms of the day but very difficult to measure accurately until clockmaking became a science in the 17th century), have become the basic units of physics. In scientific work it is usual to employ the cgs system, but the present-day tendency is for this to be replaced by the mks system. As science advances, the various arbitrary units are gradually being replaced by derived units based on the cgs or mks system; and thus the calorie and the various arbitrary viscosity units, for example, are becoming obsolescent at the present time. The arbitrary standards of the cgs and mks systems are in the form of blocks of metal, and their use has shown that nature itself has precise standards which would form a more easily reproducible system of units. Thus the mass of the proton is a fundamental unit of mass and the wavelength of the cadmium red line is a unit of length which is completely reproducible. Atomic constants of various kinds like charge, mass, action, etc., provide the fundamental and absolute units of physics, as all matter is built up from these units; and so, if science could start afresh, these would probably form the basis of a system of units. However, this fresh start is hardly likely; and so the most probable change in units will be to redefine the present units in terms of atomic quantities.

Although the general principles of the relationships between units are easy to establish, it is not easy to realize these relations accurately by experiment or to calibrate substandards of these units for use in the ordinary laboratories of the world. During the past century, many of the most distinguished physicists, including such notable men as Joule, Maxwell, Rayleigh, and Michelson, collaborated in this task. The determination of a conversion factor is one of the most difficult investigations in physics, as it demands the maximum accuracy and a scrupulous attention to detail in order that stray errors may be eliminated. Today such investigations are almost entirely performed by the various national standardizing laboratories such as the National Physical Laboratory in England, the National Bureau of Standards in the United States, and the *Bureau des Poids et des Mesures*

in France. These institutions also undertake the calibration of the various physical instruments in order that their scale readings may be converted to absolute values. As this is an expensive process, most laboratory instruments are calibrated by their makers by comparison with other instruments which they have had calibrated at the standardizing institute.

Nearly all measurements made in physics are made with an instrument calibrated in terms of some unit and therefore consist of two parts, a magnitude of the pointer reading and the unit in which that instrument is calibrated. It is essential that both the magnitude and the unit should be recorded in the experimental account. One of the commonest mistakes in student work is to use one unit in the actual measurement and to assume that a different unit was used when the result is calculated;[1] and, even apart from this, it would seem good physics to record a physical measurement in such a way that it was clearly distinguishable from a pure number and from other types of measurement.

3. Errors in the Measurement of a Single Physical Quantity

Every measurement is subject to limitations, either because the observer's senses are limited in resolving power or because there is a physical limit to the sensitivity of the instrument. No observer can detect changes in the position of a pointer less than a certain amount without the aid of additional devices such as magnifiers or vernier scales; and even these will not decrease the limit indefinitely, owing to their own limitations. To illustrate this point, it is not possible to read a meter rule to less than 0.1 mm without external aid; and it is doubtful whether it would be possible to improve this to 0.01 mm with the aid of a magnifier, in view of the width of the division marks and the difficulty of aligning the rule with the object to be measured. Modern developments have shown that Heisenberg's uncertainty principle sets a limit to all physical measurements, but this is somewhat academic as far as most scientific instruments are concerned because other factors prevent this accuracy being obtained. Of these factors, the most universal is the onset of brownian movement, e.g., in a moving-coil galvanometer; but, for each class of instrument, there are factors which are peculiar to that type of instrument (such as the limit for mercury-in-glass thermometers set by the changes in bulb size).

It is surprisingly easy to misinterpret readings of the same physical quantity obtained with different measuring instruments by assuming that the two scale readings should be directly compared. For example, two observers A and B measure two currents I_a and I_b with different ammeters and quote their results as 2.35 and 2.46 amp, respectively. What, in fact, do we know about these currents? The immediate suggestion is that I_b is greater than I_a by 0.11 amp, but this is based on the sweeping assumption that each ammeter reads the true value of the current passing through it. In fact, the meters may have been incorrectly calibrated and the readings may need to be corrected to, say, $I_a = 2.37$ and $I_b = 2.42$ amp, while a consideration of the accuracy obtainable with these instruments might indicate that I_a is uncertain by ± 0.03 amp and I_b by ± 0.04 amp. This means that A knows his current lies between 2.34 and 2.40 amp, while I_b lies between 2.38 and 2.46 amp. If this were the case, it would not be possible to distinguish between the two currents using these measurements. This example illustrates the caution required in interpreting physical measurements. No measurement is ever perfect, and to find the significance of a given measurement it is essential to examine the causes of imperfection in detail. These imperfections can be divided into three classes, *inherent error*, *reading limit*, and *random error;* and these will be considered in turn.

Before doing this, it is as well to draw attention to the fact that any pointer-reading measurement really involves two readings, *viz.*, the actual reading and the zero-point reading of the scale. It is obviously not possible to read the zero of the scale any more accurately than any other point, and it is even possible to forget to check whether the actual zero of the instrument corresponds with the zero of the scale. Even when

[1] In the author's experience, this type of mistake is most common in the various experiments on elasticity and few students seem able to perform the complete set in the laboratory course without failing to convert some unit, e.g., load in kg to dynes, or the diameter of a wire in mm to cm.

the zero point is checked, there is a natural tendency to ignore a very small error; this type of error is avoided when all readings are made as differences.

The *inherent error* of an instrument is present in all readings made with the instrument and it may arise either because the instrument is incorrectly set up (e.g., a tangent galvanometer may be incorrectly orientated with respect to the magnetic meridian), or has been incorrectly calibrated. The first type of error can usually be made negligible by careful attention to the setting-up procedure or by using a suitable experimental technique to reduce the effect (e.g., the use of the dip circle), while the second type may be overcome by having the instrument calibrated at a standardizing institute. It is very easy to overlook this need for calibration, and the literature of science contains many examples of a revision of reported experimental values because of the later discovery that an instrument was incorrectly calibrated; it would be invidious to quote examples where the experimenter could be blamed for his failure to calibrate his apparatus correctly. Inherent errors in measurements are very difficult to detect and are eliminated only by that thorough and careful attention to detail which distinguishes the great experimenter. In the rest of this paper, it will usually be assumed that this kind of error has been eliminated.

The position of a pointer with respect to a scale can be read only to a certain limit which is hereafter called the *reading limit*. This term is not standard nomenclature and has been adopted because the writer is not aware of a suitable word already in use with this connotation. In the mind of students there appears to be a complete confusion between random error and the reading limit, and it seems desirable to introduce yet another technical term in order to clarify this situation. The reading limit of an instrument means that there is a physical limit to the accuracy with which the position of the pointer may be determined on the scale. All experimenters will agree that such a limit exists, but they may not agree as to its value; thus, a first-year student will usually say that it is only possible to read a meter stick to 1 mm, but Searle expected his first-year students to record lengths with a meter stick to 0.1 mm.[2] For most instruments, practice will make the reading limit smaller—in fact, in most cases it does become possible to estimate a tenth of a division—and it is very desirable that every student should train himself to make the best use of the commoner instruments of physics. It is clear that there is little point in using a meter stick to distinguish between objects of the order of a millimeter in length, because the objects themselves are but slightly longer than the reading limit. The reading limit of an instrument is a fundamental factor in deciding the kind of measurement for which an instrument is suitable. It must be mentioned that the more subjective types of instruments like photometers also have reading limits, but that it is much more difficult to give them a quantitative value and they vary widely with the observer.

The last source of error to be considered is the *random error*. It is never possible to duplicate precisely the conditions under which an experiment is made, since all the circumstances on which the measurement depends are never precisely known and so cannot be repeated. Thus when a measurement is repeated under conditions apparently identical, the minor factors will probably differ and so it is to be expected that the result may be slightly different. As the error cannot be attributed to any one cause, it is usually called the random error. Sometimes all the readings seem to agree, but this means only that the random error is smaller than the reading limit. As the accuracy of measurement is increased, the random error becomes of increasing importance; and when it becomes much greater than the reading limit, there is no point in trying to improve the reading limit further. Very few of the ordinary physical measurements exhibit random error to a marked degree, but it can frequently be detected in the deflection of a ballistic galvanometer. An analogous effect, more frequently encountered, is a random variation in the quantity measured, e.g., the diameter of a uniform wire; and, if the effect is truly random, it may be regarded as a random error in the measurement and treated as such. The effect is

[2] This is the personal experience of the author and is also illustrated in Searle's well-known textbooks of practical physics, e.g., *Experimental Optics* (Cambridge University Press, London, 1925).

truly random only if the variation does not show any systematic trend; thus a "uniform wire" cannot be regarded as showing a random error if it is really a truncated cone.

4. The Best Value of a Measured Quantity

As all measured quantities suffer from the limitations discussed in the previous section, it becomes necessary to consider the significance of a measured quantity. It has been shown that all measurements are uncertain by at least the reading limit; and, where random error is important, the uncertainty may be considerably greater. In scientific work, the aim is to state precisely what a measurement means; hence a value of the measurement must be quoted with an indication of its reliability.

Where random error can be ignored, only one value is obtained by repeated measurements; and, obviously, this is the value to quote. As the uncertainty of the value in such a case is represented by the reading limit, this may be used to indicate its reliability; thus, a length may be measured with a meter stick as 14.62 cm, and this could be quoted as 14.62 ± 0.01 cm if the reading limit is taken as 0.01 cm.

If the measurement has a random variation, the problem is more complex, as it is necessary to find both the best value and a method of representing the reliability. This is one of the basic problems of statistics and has been adequately treated in the literature.[3] All that is required for the present purpose is to state that the mean value is the best value and that the reliability can be expressed in terms of the standard deviation of the mean. In the older literature of science, it was common to use the *probable error* as a measure of reliability; but there does not appear to be any good reason for its use, as it is, in fact, normally calculated from the *standard deviation*. The standard deviation of the mean has two important properties as an indication of reliability; namely, it is inversely proportional to the square root of the number of observations, and there is only 0.3 percent chance of the true value lying farther from the mean value than plus or minus three times the standard deviation. This means that to increase the reliability of the best value more observations have to be taken but that this process ceases to be economical after about 20 observations have been taken, as the narrowing of the error proceeds very slowly thereafter. As an example, ten observations of the deflection of a ballistic galvanometer when a condenser is discharged might yield a mean value of 16.3 cm and the standard deviation of the mean might be 0.3 cm and this would be quoted as 16.3 ± 0.3 cm. If fewer than 6–10 observations are made, it is difficult to estimate the standard deviation, owing to the small number of observations; and so it will be safest to regard the maximum deviation as the estimate of reliability.

[3] For example, Whittaker and Robinson, *The Calculus of Observations* (Blackie and Son, Ltd., London, 1924) is an admirable book on all questions concerning the mathematical treatment of experimental results in physics.

PART II. A PHYSICAL EXPERIMENT

1. The Nature of an Experiment

So far only *single* physical measurements have been considered; now, it is necessary to proceed a stage further and to examine the use that can be made of physical measurements. Measurements are made only for some specific purpose; and, in physics, the purpose of all measurements is to examine the nature of the universe. Man is an essentially curious animal and has always been interested in his environment; he has asked questions about his surroundings, and the true scientist has always endeavored to answer these questions by the experimental approach. Theory should always be the handmaiden of experiment, and the validity of any theory must be decided by the way in which its conclusions are supported by experimental evidence. In decadent times (scientifically), man has tried to short-cut the labor of experiment and to answer his questions by the use of intellect alone; this labor-saving effort has led to the growth of many false ideas and has always retarded the progress of science.

What sort of question has man asked about his environment? The most obvious kind of questions can be called "why" questions. "Why is the sun brighter than the moon?" "Why is glass transparent, but not Bakelite?" "Why does wood burn, but not iron?" "Why is water liquid and iron solid at room temperature?" This sort of question was first asked aeons ago; and not one of them has ever been *really* answered, for every

answer can be converted into another "why" question. For example, the answer to the last question might be given as "Because the melting point of water is much lower than that of iron." This answer can be immediately converted to "Why is the melting point of water much lower than that of iron?" If the latter question were answered, it could in turn be converted into yet another "why" question and the process repeated. A "why" question essentially asks for the reason for the universe being as it is, and this information is inaccessible to scientific inquiry. Instead, the scientist must take the universe "as he finds it" and ask questions about the way in whlch it "works"; i.e., he asks "how" questions. "How much brighter is the sun than the moon?" "How much light is transmitted through glass?" "How does wood burn?" "How does a liquid differ from a solid?" This change in the type of question represents a fundamental limitation to the realm in which science has a right to be heard; an adequate discussion of its significance would require a deep philosophical investigation which would be out of place in the present article.

In the scientists' efforts to answer "how" questions, two kinds of investigation have developed: first, the more important kind of investigation, is the discovery of the relationships between observed physical quantities, and, second, is the determination of the constants in such relations which characterize the different physical substances. Thus, investigations on the elastic properties of materials led to the discovery of Hooke's law; and, now that the law is known, much of the experimental work in elasticity is concerned with the investigation of the elastic moduli of different materials in order that the general relationship may be useful in both science and industry. Although there are these two types of investigation, the general experimental method is equally applicable to both and the purpose of a university course in experimental physics should be to train a student to be able to plan any experimental investigation he may find it necessary to perform. Unfortunately, most courses seem to be designed rather to repeat classical experiments in a simplified form than to train a student in the general principles of experimental technique; all too often students appear unable to transfer the technique points of any one experiment to other experiments.

Besides the limitations on each measurement in an experiment already discussed, it is usually necessary to assume some theory to calculate the result of an experiment; and if this theory is later discarded, the results will have to be recomputed in terms of the new theory. An illustration is found in the mistaken assumption used by Rowland in determining the wavelength of the standard spectral lines. The use of incorrect theory is difficult to detect and behaves in a similar manner to the more usual problem of systematic error which is to be discussed.

Nature never provides a simple occurrence, and so the simple relationships are obscured by subsidiary effects; in the laboratory, it is customary to try to design experiments so that the number of variables is reduced to a minimum (usually two), in order to insure that the effect is produced by a single cause. Although the attempt is made, it never succeeds perfectly; and so various sources of error traceable to incorrect assumptions can creep in. Some examples are

(*i*) The temperature is assumed constant during applications of Boyle's law.

(*ii*) It is assumed that there is no heat loss from the surface of the bar in Searle's method[4] for thermal conductivity.

(*iii*) It is assumed that there is no friction at the pulley of a sonometer.

(*iv*) Perfection is assumed in measuring instruments; that a voltmeter takes no current; that an ammeter has no resistance; that a thermometer has no thermal capacity; that the reading of a ballistic galvanometer is independent of the time constant of the circuit.

It would be quite impossible to make any progress in science without making such assumptions, but it is very desirable to conduct subsidiary investigations to find out how valid they are. These assumptions lead to two kinds of error, systematic and random. If a factor enters into the results of any experiment and no allowance has been made for it, then the results will show a systematic error. In a calorimetry experiment, for example, the thermal capacity of the thermometer may be ignored and then the calculated heat change will be systematically wrong; that is, it will always be either too low

[4] Phil. Mag. 9, 125–129 (1905).

or too high according as the thermometer is cooled or heated. In electricity, the assumption that a voltmeter takes no current produces a similar systematic error.

It has already been shown that a single measurement is liable to a random error; as an experiment is a series of measurements, the experimental data will suffer a random error from this cause. The more serious type of random error in some cases is due to random causes in the experiment itself rather than in single readings; for example in a heat experiment, the effect of drafts will probably cause a fluctuation between different experiments.

To eliminate a systematic source of error, it is necessary to know that it exists. This appears to be a statement of the obvious, but it is almost impossible to list all the conceivable causes of systematic error in any one experiment. The more obvious ones are carefully investigated, and it is often possible to make corrections for them. But what of the unexpected and undetected sources of error? How are these to be eliminated? The only safe way of keeping systematic error to a minimum is to determine the quantity by several methods, differing from one another as widely as possible: if the results agree, then systematic errors can be assumed to be reasonably small. A classic example of this is the determination of the electronic charge: Millikan determined it first by his oil-drop method, but later work with x-rays has shown that his value was in error by a far larger quantity than his experimental data suggested. Many errors of a systematic nature can be expected, and, in these cases, it is usually possible to estimate the magnitude of the effect—the so-called "correction"; this problem will be discussed in Sec. 7. In designing an experiment, it is desirable that all systematic errors should be either negligible or capable of correction; and, in some cases, it is even worth while to replace an uncalculable systematic error by a larger but calculable one.

The effect of the additional source of random error due to the experiment rather than the individual measurements is to make it desirable to calculate the mean value and standard deviation of the final answer directly rather than by compounding those for the individual measurements. For those investigations in which the results take the form of a functional relationship, as in Boyle's law, it is usual to use graphical methods rather than computation; for the highest accuracy, numerical methods must be resorted to and the method of least squares used.[3] Wherever possible the graph should be made to take the form of a straight line, as it is the easiest graph to draw accurately and also to interpret.

2. Corrections

Many of the systematic errors can be allowed for by making additional assumptions and increasing the number of experimental data. Thus Regnault's cooling correction for the loss of heat in a calorimetric reaction requires measurements of the rate of cooling and must also assume that the rate of loss of heat obeys Newton's law. In this case, it is extremely difficult to estimate the cooling correction accurately, but this is not serious if the correction is small compared with the recorded temperature change; if the correction is 0.1°C and the temperature rise is 5°C, a 50 percent error in the cooling correction is only a 1 percent error in the total temperature rise.

To evaluate any correction, it is necessary to assume a relationship between measured quantities from which the correction may be computed; it is usually very difficult to be sure that the relation is correct, and normally the correction has an appreciable error. In these circumstances it is very desirable to keep the correction as small as possible, and so every effort must be made in designing an experiment to eliminate all unnecessary corrections and to keep others very small by the right choice of method and apparatus. In planning an experiment, therefore, it is advisable to list possible sources of systematic errors, to consider whether corrections can be applied, and, if possible, to estimate their probable magnitudes.

3. Preliminary Experiments

In the last section, it was shown that much care is required in planning an experiment if the best possible results are to be obtained. It is extremely difficult to do this planning unless the investigator has a wide knowledge of the problems likely to be met with in that particular field of physics and, in particular, has an ade-

quate knowledge of the order of magnitude of the quantities to be measured. In all experiments it is necessary to adjust conditions that the changes occurring can be measured to the desired accuracy with the instruments available. For example, to determine the heat of fusion of ice it is necessary to adjust the ratio of the mixture of ice and water that the temperature change is suitable for measurement with the thermometer available. Knowing that the heat of fusion of ice is approximately 80 cal/g, it is easy to calculate the water-ice ratio so that the final accuracy shall be of the order of 1 percent, using a 1/10°C thermometer; for the temperature change to be measured to 1 percent accuracy it must be at least 10°C, and so the weight of water must be of the order of 8–10 times the weight of ice.

If the heat of fusion of ice were unknown, it would not be possible to perform this type of calculation: how then are we to find the right instruments and conditions under these circumstances? The only way is by making preliminary experiments to determine the conditions required in the accurate experiments. These "trial experiments" not only permit the right instruments to be chosen but also show up some of the experimental difficulties which will be met in the investigation proper. It is interesting to note that this procedure is precisely that adopted by industry when a new product or process is being developed, and in both cases it is a sure method of saving both time and money.

Preliminary experiments are designed to save time in carrying out the main investigation and so need not be performed to a high standard of accuracy. In most cases a 50 percent error in a quantity to be measured will not affect the form of apparatus which is most suitable. Frequently it is not even necessary to make the preliminary experiment carefully quantitative. A trial experiment on the heat of fusion of ice can be made by dropping a lump of ice into a beaker of water and noting the change in temperature.

4. Null Methods

In many branches of physics such as magnetometry, resistance measurement, photometry, ac bridges, it is possible to use null methods, but they are open to the objection that they are only comparative methods. To obtain absolute values, one of the quantities to be compared must be known in absolute units; and the accuracy with which it is known determines the maximum accuracy with which the other can be determined. As discussed earlier in Sec. 3, in these days secondary standards, accurately calibrated by national standardizing institutes, are easily available; and so this objection has, in reality, little validity. It is probably true to say that some of the most accurate experiments in a teaching course are null methods; the great advantage of null methods is that the observer has merely to detect a balance and so does not require to know the law of the instrument used as detector. It is also a fact that with many instruments it is possible to detect a smaller departure from the balance position than the smallest change in deflection which could be measured in the ordinary use of the instrument. By using proper precautions, it is usually possible to reduce the systematic errors in null methods to a smaller magnitude than in the corresponding deflection method, as it is not necessary to make as many assumptions. In a Wheatstone bridge method neither the law of the galvanometer nor the behavior of the battery need be known. In null methods it is often possible to reduce other systematic errors by such processes as the reversal of the position of the two things being compared and by similar processes; examples are

(*i*) Reversal of the current in a bridge to eliminate thermal emf's.

(*ii*) Compensating leads in a platinum resistance thermometer.

(*iii*) Reversal of the magnets using a deflection magnetometer as a null instrument in order to eliminate errors due to magnetic asymmetry of the magnets.

5. Experimental Error

In Sec. 6, it has been suggested that the standard deviation of the final result should be calculated from the actual experimental results rather than from the standard deviations of the individual readings, owing to the possible existence of errors other than those in the measurements. When, however, an investigation is started, it is impossible to predict what the standard deviation will be; but the investigator re-

quires some idea of how accurately each measurement must be made in order that, assuming the random error is not large, he may achieve the desired accuracy in the final result. For a single measurement, it was seen that the reading limit provided an indication of the accuracy of a reading; and it is now desirable to define a new quantity, *the experimental limit*, to give a similar indication for a complete experiment. Clearly, the accuracy of the final result must depend on the accuracy of all the readings used to obtain that result, and so the reading limits of all the measurements are "propagated" into the experimental limit. Like the reading limit, the experimental limit does not appear to have been named before, and so the precise definition is a matter of the author's personal choice. In elementary calculus, it is usual to derive the effect of the combination of errors on a formula when partial differentiation is discussed; and it is proposed that these formulas are applied to the reading limits of the various measurements in order to find the experimental limit, with the additional restriction that, as is usual with standard deviations, it is the squares which are added. To clarify this definition consider the case $z=uv$, where u and v have reading limits du and dv; then by the rules of partial differentiation the experimental limit, dz, is given by $dz/z=(du/u)+dv/v$ if we do not apply any additional restriction (which really means that errors are assumed independent) and becomes the true experimental limit if $(dz/z)^2=(du/u)^2+(dv/v)^2$. This definition of experimental limit has two features worthy of note and further discussion; first, ignoring the susceptability of the individual readings to random error and, second, the addition of the squares rather than the reading limit. Neither is necessary; but the author believes that in this way it is possible to define a quantity easy to calculate and free from serious ambiguity, which gives a reasonable estimate of the likely accuracy of any experimental arrangement contemplated unless serious sources of systematic and random error associated with the particular experiment occur.

Clearly it is very difficult to calculate the experimental limit until approximate values of the quantities to be measured are known. The calculation can be made with useful accuracy, however, from data obtained from preliminary experiments. It should never be necessary to calculate the experimental limit to high accuracy, as its real use is to determine the order of magnitude of the expected error in the final result; a factor of two will rarely be serious, owing to random and systematic errors in the experiment. In theory, it is always possible to reduce the random error below the experimental limit; but this is usually impracticable, owing to the very large number of experimental determinations required.

In order to study how the reading limits of the individual readings affect the experimental limit it is easiest to consider typical examples of physical experiments. An excellent example for student use is the method of mixtures in calorimetry, and a single example of this method has been successfully used by the author to draw the attention of students to many of the points discussed subsequently.

(*i*) The reading limit of one or more readings may predominate in determining the experimental limit; this is especially true when one measurement can be made only to a very low standard of accuracy. In such cases, it is permissible—even desirable—to make some of the other measurements to a lower order of accuracy than the instruments are capable of, as this will not affect the experimental error appreciably and will save time. For example, in calorimetry the temperature rise is often small and can be determined in an elementary laboratory only to a few percent, and it may well happen that masses need not be measured beyond ± 0.1 g.

(*ii*) If a small difference between two large quantities is involved, these quantities must be known very accurately if the experimental limit is to be small. A good example is the measurement of the mass of copper deposited in the determination of the electrochemical equivalent of copper. Typical values of the mass of the cathode before and after the experiment are 53.214 and 53.349 g with a reading limit of, say, 0.001 g. The experimental limit in the deposited mass is 1 percent yet the reading limit is 0.002 percent for each weighing.

(*iii*) It is often overlooked that values of physical constants taken from tables are accurate only to a limited number of figures and must be treated in the same way as other measure-

ments when estimating the experimental limit, the reading limit being replaced by the quoted accuracy. In work of the highest precision, it is often essential to determine these constants afresh, as the tabulated values prove to be not sufficiently accurate; for such reasons Millikan had to perform experiments on the viscosity of air when he determined the electronic charge. In a similar way, any apparatus used in an experiment whose value is assumed known, such as the resistance of a resistance box or the volume of a pipette, will have an equivalent reading limit.

(*iv*) Errors are magnified if a measured quantity is raised to a power and diminished if a root is taken; this is especially important in viscosity and rigidity measurements where small radii have to be measured and are then raised to a fourth power.

The experimental limit gives useful information about the number of significant figures to be quoted in the final result. Thus in a specific heat problem the experimental data might yield a numerical value of 0.073423 . . . cal/g°C; if the experimental limit is 3 percent, then this is equivalent to 0.002 cal/g°C, and so it is of no significance to quote more than two significant figures. In student work it is rarely possible to take sufficient readings to be able to calculate the standard deviation of the mean, and the experimental limit is thus the only indication of the probable accuracy of the work. It may also be used to improve the over-all design of the experiment by indicating where the accuracy is least.

SUMMARY

1. All measurements are liable to error.
2. Inherent errors in measurements can be overcome by calibration.
3. The limit on any measurement is the reading limit.
4. In either single measurements or experiments, results will be affected by random error in addition to inherent errors and either the reading or experimental limit.
5. The reading limits of the measurements made in an experiment set the limit to the experimental limit (i.e., the possible accuracy of the final result).
6. Various stray effects in an experiment can often be corrected for, but they should always be kept as small as possible.
7. In all experimental work, the experimental limit should be calculated and also, where possible, the standard deviation of the mean value. These data should be used to determine the number of significant figures in the final result.
8. Most measurements and results have units which should always be recorded.
9. All investigations which involve unknown magnitudes should commence with a preliminary experiment to determine the orders of magnitudes involved, but too much time and effort should not be used in such an experiment in order to achieve high accuracy.
10. In planning a detailed investigation, the experimental limit for any proposed method should be calculated and used to decide whether any measurements are being planned for too great precision and whether the final result would be materially improved by altering the reading limit of any measurement.

ACKNOWLEDGMENTS

The author is indebted to all who have taught him physics for awakening an interest in underlying assumptions of scientific work; but, in particular, Mr. W. H. Crabbe, Mr. E. W. Chanter, Dr. G. F. C. Searle, and the late Sir Arthur S. Eddington have influenced his thought on this subject. The duplicated Supplement in which these ideas were first expressed was edited and largely written by a colleague, Dr. J. I. Jones, without whose encouragement and helpful discussion this work would never have been undertaken.

Real progress in any science involves an awareness of our assumptive worlds, a consciousness of their inadequacy, and a constant self-conscious attempt to change them so that the intellectual abstractions they contain will achieve increasing breadth and usefulness. Real progress in science means much more than adding to existing knowledge.—HADLEY CANTRIL *et al.* Science **110**, 462 (1949).

One- and Two-Dimensional X-Ray Diffraction

B. D. CULLITY
Department of Metallurgy, University of Notre Dame, Notre Dame, Indiana
(Received January 8, 1951)

A description is given of the diffraction phenomena from crystals so small in some directions that they may be considered essentially as one- or two-dimensional crystals. The basic theory of x-ray diffraction, in terms of the Ewald construction in the reciprocal lattice, is first presented for the case of a three-dimensional crystal and then extended to crystals consisting only of a plane of atoms one layer thick or of a linear chain of atoms. Intermediate cases are then considered and experimental methods of observing these phenomena are discussed. Finally, examples are given of one- and two-dimensional crystals which have been found in layer lattices, fibrous materials, supersaturated metallic solid solutions, and metal crystals having stacking errors.

CRYSTAL structures are seldom perfect, although many are perfect enough to give marvellously clear-cut diffraction in agreement with the Bragg law or the equivalent set of Laue equations.

The majority of crystals, however, have imperfections of the following nature: thermal vibration of the atoms, bent and twisted lattice planes, irregularities in atomic spacings, vacant lattice sites, etc. Imperfections of the grosser sort become evident on a diffraction pattern made by any of the usual methods, while certain types of imperfections can only be disclosed by careful design of the experimental conditions. The practical importance of such studies is due to the fact that many seemingly minor crystal imperfections have a marked effect on the properties of the material, whether it be metallic or nonmetallic.

Another type of imperfection which may exist is one of size or shape. The perfect three-dimensional crystal has, among other properties, infinite size. Relative to such a crystal, a two-dimensional crystal, consisting of a single layer of atoms, and a one-dimensional crystal, consisting of a single row of atoms, must be considered imperfect simply because they lack three-dimensionality. This lack is reflected in a relaxation of the conditions for diffraction so that, instead of three, only one or two of the Laue equations need be fulfilled.

The purpose of this paper is to review the basic theory underlying diffraction from one- and two-dimensional lattices, phenomena which for convenience will be termed "one- and two-dimensional diffraction." This terminology does not in any way imply that the diffracted beams are confined to one or two dimensions in space, since actually they are more numerous than the beams diffracted by a three-dimensional crystal. Because one- and two-dimensional diffraction are best considered as special cases of three-dimensional diffraction, the theory of the latter will be briefly outlined first, in a manner closely modeled on that of Guinier.[1]

THREE-DIMENSIONAL DIFFRACTION

This problem will be treated in terms of the reciprocal lattice, that very powerful concept introduced into the field of x-ray diffraction by Ewald[2] in 1921. The reciprocal lattice of a three-dimensional crystal is an array of points in space, their position being determined by repetition of the fundamental vectors $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$.[3] These vectors, which outline the unit cell of the reciprocal lattice, are defined in terms of the vectors $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, which outline the unit cell of the crystal lattice, by the following equations:

$$\left.\begin{aligned}\mathbf{b}_1&=(1/V)(\mathbf{a}_2\times\mathbf{a}_3)\\ \mathbf{b}_2&=(1/V)(\mathbf{a}_3\times\mathbf{a}_1)\\ \mathbf{b}_3&=(1/V)(\mathbf{a}_1\times\mathbf{a}_2)\end{aligned}\right\}, \tag{1}$$

where V is the volume of the unit cell of the crystal.

[1] A. Guinier, *Radiocristallographie* (Dunod, Paris, 1945).
[2] P. P. Ewald, Z. Krist. 56, 129 (1921).
[3] Vectors are here represented by boldface symbols. The same symbol in italics stands for the absolute value of the vector.

The above relations become less formidable for any crystal whose unit cell is based on mutually perpendicular vectors. In such cases, $\mathbf{b}_1$, $\mathbf{b}_2$, and $\mathbf{b}_3$ are parallel, respectively, to $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$, while b_1, b_2, and b_3 are simply the reciprocals of a_1, a_2, and a_3.

In any crystal, however, it remains true, for example, that

$$\mathbf{a}_1\cdot\mathbf{b}_1=1 \quad \text{and} \quad \mathbf{a}_1\cdot\mathbf{b}_2=0$$

or, in general,

$$\left.\begin{aligned}\mathbf{a}_m\cdot\mathbf{b}_n&=1, \quad \text{if} \quad m=n\\ &=0, \quad \text{if} \quad m\neq n\end{aligned}\right\}. \tag{2}$$

The reciprocal lattice is defined in this way in terms of the crystal lattice in order to give it the following two geometric properties:

(1) A vector $\mathbf{H}_{hkl}$ drawn from the origin of the reciprocal lattice to any point in it having coordinates hkl is perpendicular to the plane in the crystal lattice whose Miller indices are hkl. This vector is given in terms of its components by the expression

$$\mathbf{H}_{hkl}=h\mathbf{b}_1+k\mathbf{b}_2+l\mathbf{b}_3. \tag{3}$$

(2) The absolute value of the vector $\mathbf{H}_{hkl}$ is equal to the reciprocal of the spacing d of the (hkl) planes, or

$$H_{hkl}=1/d_{hkl}. \tag{4}$$

The important thing to note about these relations is that the reciprocal lattice array of points completely describes the crystal in the sense that each reciprocal lattice point is related to a set of planes in the crystal and represents the orientation and the spacing of that set of planes.

Diffraction may now be considered. Let the incident x-rays have a wavelength of λ and let the incident and diffracted beams be represented by the unit vectors $\mathbf{S}_0$ and $\mathbf{S}$ (Fig. 1). To determine under what conditions diffraction will occur, it is necessary to determine the phase difference between rays scattered by the atom O at the origin and any other atom A whose coordinates with respect to the origin are $p\mathbf{a}_1$, $q\mathbf{a}_2$, and $r\mathbf{a}_3$, where p, q, and r are integers.[4] Thus, we have

$$\mathbf{OA}=p\mathbf{a}_1+q\mathbf{a}_2+r\mathbf{a}_3.$$

[4] The crystal lattice is here assumed to be primitive. If not, the above treatment is still valid, provided each reciprocal lattice point is weighted by the appropriate value of the scattering power ($F\bar{F}$, where F is the structure factor) for the particular lattice plane concerned. Some planes may then have zero scattering power, thus eliminating some reciprocal lattice points from consideration.

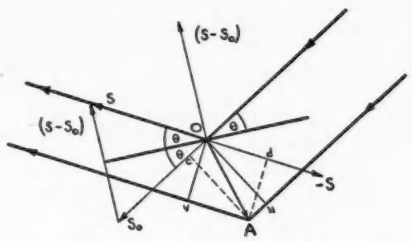

FIG. 1. Diffraction of x-rays by atoms O and A in a three-dimensional crystal.

The vectors $\mathbf{S}_0$, $\mathbf{S}$, and $\mathbf{OA}$ are, in general, not coplanar. Lines Ou and Ov are wave fronts perpendicular to the incident beam $\mathbf{S}_0$ and the diffracted beam $\mathbf{S}$, respectively. Let D be the path difference for waves scattered by O and A, then

$$\begin{aligned}D&=uA+Av\\ &=Oc+Od\\ &=\mathbf{S}_0\cdot\mathbf{OA}+(-\mathbf{S})\cdot\mathbf{OA}\\ &=-\mathbf{OA}\cdot(\mathbf{S}-\mathbf{S}_0).\end{aligned}$$

The corresponding phase difference ϕ between waves scattered by O and A is given by

$$\begin{aligned}\phi&=2\pi D/\lambda\\ &=(-2\pi/\lambda)(\mathbf{S}-\mathbf{S}_0)\cdot\mathbf{OA}.\end{aligned} \tag{5}$$

Diffraction is now related to the reciprocal lattice by expressing the vector $(\mathbf{S}-\mathbf{S}_0)/\lambda$ as a vector in the reciprocal lattice. Let

$$(\mathbf{S}-\mathbf{S}_0)/\lambda=h\mathbf{b}_1+k\mathbf{b}_2+l\mathbf{b}_3.$$

This is now in the form of a vector in reciprocal space; at this point, no particular significance is attached to the parameters h, k, and l. They are continuously variable and may assume any values, integral or nonintegral.

Equation (5) now becomes

$$\phi=-2\pi(h\mathbf{b}_1+k\mathbf{b}_2+l\mathbf{b}_3)\cdot(p\mathbf{a}_1+q\mathbf{a}_2+r\mathbf{a}_3).$$

The scalar product on the right is evaluated with the aid of Eqs. (2). The phase difference then becomes

$$\phi=-2\pi(hp+kq+lr). \tag{6}$$

A diffracted beam will only be formed if reinforcement occurs, and this requires that ϕ be an integral multiple of 2π. This can happen only if

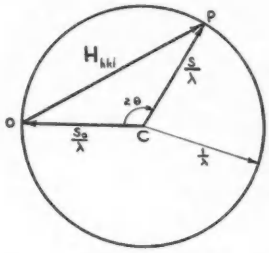

FIG. 2. Ewald construction. Section through the sphere of reflection containing the incident and diffracted beam vectors, $\mathbf{S}_0/\lambda$ and $\mathbf{S}/\lambda$, and the reciprocal lattice vector $\mathbf{H}$. Here P is the point hkl in the reciprocal lattice.

h, k, and l are integers. Thus, the condition for diffraction is that the vector $(\mathbf{S}-\mathbf{S}_0)/\lambda$ end on a *point* in the reciprocal latice, or that

$$(\mathbf{S}-\mathbf{S}_0)/\lambda=\mathbf{H}_{hkl}=h\mathbf{b}_1+k\mathbf{b}_2+l\mathbf{b}_3, \qquad (7)$$

where h, k, and l are now restricted to integral values.

Equation (7) is one way of expressing the three Laue equations in a condensed form. The Bragg law follows from it. As shown in Fig. 1, $(\mathbf{S}-\mathbf{S}_0)$ bisects the exterior angle between $\mathbf{S}$ and $\mathbf{S}_0$, and the diffracted beam $\mathbf{S}$ can thus be considered as being *reflected* from a set of planes perpendicular to $(\mathbf{S}-\mathbf{S}_0)$. The angle between $\mathbf{S}$ (or $\mathbf{S}_0$) and these planes is θ; and, since $\mathbf{S}$ and $\mathbf{S}_0$ are unit vectors, we have

$$(S-S_0)=2\sin\theta.$$

Therefore, we have

$$(2\sin\theta)/\lambda=(S-S_0)/\lambda=H\equiv 1/d$$

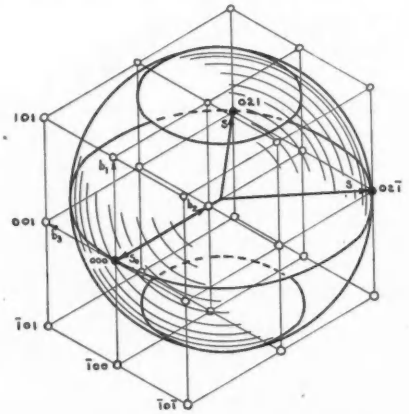

FIG. 3. Diffraction from a three-dimensional crystal. Sphere of reflection intersecting a reciprocal lattice of points. Full circles are points touching the surface of the sphere. (Here, and in Figs. 4–6, the incident and diffracted beam vectors are simply labeled $\mathbf{S}_0$ and $\mathbf{S}$, it being understood that the length of each is equal to $1/\lambda$.)

or

$$\lambda=2d\sin\theta.$$

The conditions for diffraction expressed in Eq. (7) may be represented graphically as follows (Fig. 2). The vector $\mathbf{S}_0/\lambda$ is drawn parallel to the incident beam and $1/\lambda$ in length. The terminal point O of this vector is taken as the origin of the reciprocal lattice, drawn to any convenient scale of reciprocal angstroms, and its initial point C as the center of a sphere of radius $1/\lambda$, drawn to the same scale. Then the condition for diffraction from a crystal plane (hkl) is that the point hkl in the reciprocal lattice touch the surface of the sphere, for only in this way can Eq. (7) be satisfied. For this reason, the sphere described about C is called the "sphere of reflection." The direction of the diffracted beam vector $\mathbf{S}/\lambda$ is found by joining the center of the sphere to the point on its surface which is touched by the reciprocal lattice point hkl in question.

Figure 3 is drawn for the case of a parallel beam of monochromatic x-rays incident on a fixed orthorhombic crystal along the direction $[0\bar{1}0]$. For the particular values of unit cell size, crystal position and wavelength chosen, only two reciprocal lattice points fall on the sphere of reflection; accordingly, two diffracted beams result, 021 and $02\bar{1}$, in the directions shown in the figure.

The common methods of x-ray diffraction are differentiated by the means used for bringing reciprocal lattice points in contact with the surface of the sphere of reflection. The radius of the sphere may be varied by varying the incident wavelength (Laue method) or the position of the reciprocal lattice may be varied by changes in the orientation of the crystal (rotating-crystal and powder methods).

TWO-DIMENSIONAL DIFFRACTION

The diffracting crystal, in this case, consists only of a two-dimensional net of atoms, the unit parallelogram being defined by the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. The phase difference equation equivalent to Eq. (6) above is

$$\phi=-2\pi(hp+kq). \qquad (8)$$

The condition for diffraction is that h and k be integral, and the relation

$$(\mathbf{S}-\mathbf{S}_0)/\lambda=\mathbf{H}_{hk}=h\mathbf{b}_1+k\mathbf{b}_2 \qquad (9)$$

must be satisfied. Under these conditions, l can have *any value*. The reciprocal lattice thus consists of a set of *lines*, or rods, perpendicular to the plane of the two-dimensional crystal. Diffraction can occur if the vector $(\mathbf{S}-\mathbf{S}_0)/\lambda$ ends anywhere on one of these lines in the reciprocal lattice, since only two components of this vector, namely, $h\mathbf{b}_1$ and $k\mathbf{b}_2$, are restricted. Thus, whenever any part of a reciprocal lattice rod touches the surface of the sphere of reflection, a diffracted beam will be formed. Figure 4 shows a reciprocal lattice of this kind and the eight diffracted beams formed when the sphere of reflection intersects a reciprocal lattice rod. The incident beam is directed along the $\mathbf{a}_2$ axis of the two-dimensional crystal.

There is an important difference between the two kinds of diffraction illustrated in Figs. 3 and 4. If the orientation of the three-dimensional crystal is changed slightly from that shown, no diffraction will occur. The two-dimensional crystal, however, can be oscillated through any angle and diffraction will still occur, since some part of the reciprocal lattice rods will still intersect the sphere of reflection. In such a case, the diffraction pattern on the film would consist of lines rather than spots.

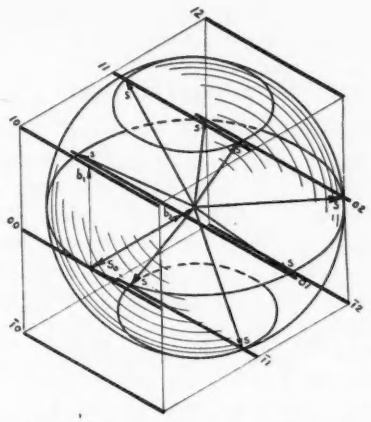

FIG. 4. Diffraction from a two-dimensional crystal having a rectangular "unit cell." Sphere of reflection intersecting a reciprocal lattice of lines. (The reciprocal lattice is composed of the heavy lines shown; the thin lines are drawn only as aids to visualization.)

ONE-DIMENSIONAL DIFFRACTION

The one-dimensional crystal is defined simply by one vector $\mathbf{a}_1$, whose absolute value is the spacing between the atoms along the row. The phase difference equation becomes

$$\phi = -2\pi(hp); \tag{10}$$

h must now be integral for diffraction to occur, while k and l can have *any value*. The necessary condition for diffraction is that

$$(\mathbf{S}-\mathbf{S}_0)/\lambda = \mathbf{H}_h = h\mathbf{b}_1. \tag{11}$$

A diffracted beam will be formed whenever the vector $(\mathbf{S}-\mathbf{S}_0)/\lambda$ ends anywhere on a *plane* in the reciprocal lattice, since Eq. (11) restricts only one component of this vector. The reciprocal lattice thus consists of a set of planes perpendicular to the row of atoms forming the crystal.

For any particular choice of crystal orientation and incident beam wavelength, cones of diffracted beams will be formed by a one-dimensional crystal, since the reciprocal lattice planes will intersect the reflection sphere in circles. This is illustrated by Fig. 5 which shows the diffracted beams formed when the incident beam is normal to the row of atoms. If a three-dimensional crystal is rotated around an axis along which it has a periodicity of d angstroms and if it is illuminated by monochromatic x-rays normal to this axis, then the diffraction pattern on a cylindrical film will consist of spots arranged on layer lines; the equivalent experiment performed with a *fixed* one-dimensional crystal of spacing d angstroms will result in a pattern of

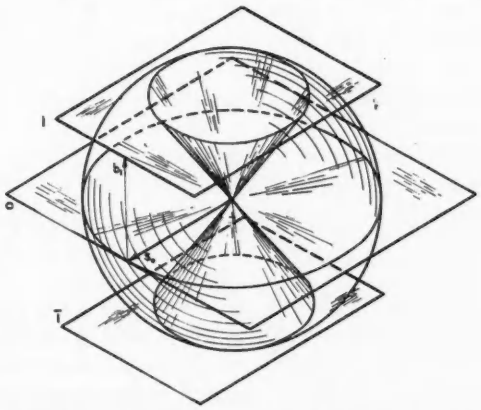

FIG. 5. Diffraction from a one-dimensional crystal. Sphere of reflection intersecting a reciprocal lattice of planes.

lines joining those points on the film where the previous spots were located.

INTERMEDIATE CASES

The above discussion refers to ideal cases where one, or two, of the three dimensions which define the crystal volume becomes equal to one atomic diameter. Actual examples of these conditions are rare, so that most interest attaches to the intermediate cases where one or more of these dimensions becomes very small but not monatomic.

A three-dimensional crystal can be imagined as becoming thinner and thinner until it consists only of a plane network one atom thick. While this is happening, every reciprocal lattice point becomes extended further and further towards its neighbors in the dimension in which the crystal is becoming thinner, until ultimately, when the crystal becomes one atom thick, these extended points join one another and form a set of lines. If the two-dimensional crystal then shrinks in one direction until it becomes a row of atoms, the reciprocal lattice lines will extend sideways until they join and become planes. Finally, shortening of the row of atoms to a point where it consists only of one atom is accompanied by a thickening of the reciprocal lattice planes until the reciprocal lattice consists simply of an extended volume in reciprocal space. (From the viewpoint of diffraction, this latter case corresponds to the scattering of x-rays in all directions by a single atom.)

If a three-dimensional crystal shrinks in size to become, not a single layer of atoms, but only a thin plate, say 10 atoms thick, then each reciprocal lattice point becomes extended only partially towards its neighbors. According to Ewald,[5] the amount of this extension is roughly $1/N$ of the distance between reciprocal lattice points in a direction normal to the thin plate, where N is the number of atom layers in the plate.

Very small crystals, therefore, have reciprocal lattices composed, not of points, but of regions of varying sizes and shapes which are reciprocally related to the size and shape of the crystal, as summarized in the table below:

Shape of small crystal	Shape of reciprocal lattice regions
plate-like	rod-like
rod-like	plate-like
equiaxed	equiaxed.

The reciprocal lattice point always extends itself in the same dimension as that in which the crystal size decreases. But the amount of the extension, as Ewald's relation shows, is very small. A crystal dimension must be reduced below about 1000 angstroms, say 300 atom layers, before the broadening of a reciprocal lattice point, line, or plane becomes experimentally observable; and even then refined experimental technique is necessary. On the other hand, it should be noted that the reciprocal lattice of any finite three-dimensional crystal does not consist of mathematical points but rather of regions whose size is inversely related to the size of the crystal.

The ordinary line broadening found on Debye patterns, owing to small particle size, can readily be explained in terms of the above concepts. If the small particles are approximately equiaxed, then the reciprocal lattice will consist of a set of small, roughly spherical regions. These small regions will be intersected by the reflection sphere in small areas, rather than points. The diffracted beam vector $\mathbf{S}/\lambda$ may end anywhere within one of these areas: there is thus a slight angular divergence of the diffracted beam and a broadening of the diffraction line on the film.

The diffracting regions in the reciprocal lattice of a thin crystal plate of infinite extent consist of a set of line segments whose lengths are inversely proportional to the thickness of the plate. (A portion of such a lattice is shown in Figs. 6(a) and (b).) However, if the lateral dimensions of the plate are quite small, the lines of the reciprocal lattice will thicken into cylinders or prisms of a particular shape, depending on the shape of the plate. Similarly, the reciprocal lattice of a small crystal rod consists of planar regions whose extent depends inversely on the thickness of the rod and whose thickness depends, also inversely, on the length of the rod.

EXPERIMENTAL OBSERVATIONS

Exploration of the diffracting regions in the reciprocal lattice can thus lead to a determina-

[5] P. P. Ewald, Proc. Phys. Soc. (London) 52, 167 (1940).

tion of the size and shape of small crystals. Essentially, this is accomplished by altering the relative positions of the reciprocal lattice and the reflection sphere and observing the diffraction effects produced.

The Laue method may be used for this purpose, as shown in Fig. 6(a). Here the range of wavelengths present in the incident beam is represented by a series of reflection spheres of continuously varying radius, some of which are sketched. A diffracted 110 streak will be registered on the film because of the rod at 110 in the reciprocal lattice. The length and width of this streak will depend on the length and thickness of the rod, respectively, provided the range of wavelengths present is large enough so that the entire length of the rod is cut by the various reflection spheres. The length and thickness of the reciprocal lattice rod in turn depend, respectively, on the thickness and lateral dimensions of the thin crystal plate causing the diffraction streak. The sphere of largest radius in Fig. 6(a) corresponds to the short wavelength limit of the continuous spectrum; the radius of the smallest sphere is not quite so definite, but is usually taken as the reciprocal of the wavelength of the K absorption edge of the silver in the film.

Strictly monochromatic radiation is generally more useful in exploring these reciprocal lattice regions. In Fig. 6(b), monochromatic radiation has been used and the crystal oscillated through the angle α with respect to the beam in order to cover the whole length of the rod at 110. The region at 000 is most conveniently investigated by making the sphere of reflection initially tangent to a reciprocal lattice rod, the angle α then being varied in steps or continuously by oscillation (Fig. 6(c)). Even with the reflection sphere tangent to the reciprocal lattice rod, diffraction streaks radiating from the central spot will be formed if the rod has appreciable thickness or if the primary beam is appreciably nonparallel.

Because of their small size, it is obvious that essentially one- or two-dimensional crystals do not exist as single crystals susceptible of being mounted in a diffraction camera. One way in which such crystals can exist is through a lack of periodicity in certain directions in a large crystal. For example, if a crystal has a layer

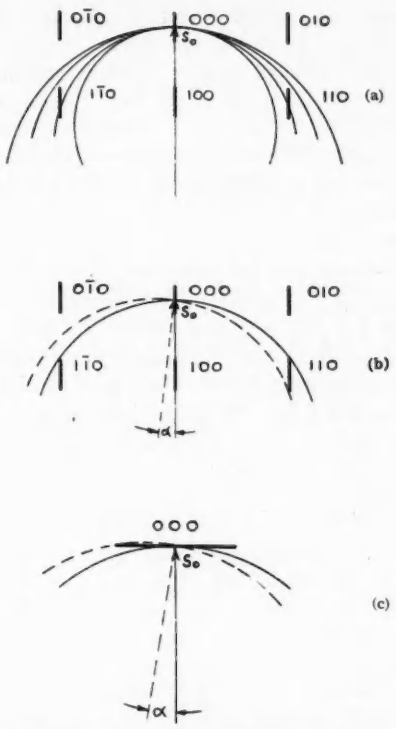

FIG. 6. Sphere of reflection and reciprocal lattice of thin crystal. Section through $l=0$ layer. (a) White radiation. (b) and (c) Monochromatic radiation, crystal oscillated through the angle α.

structure in which each layer possesses a perfect arrangement of atoms but in which there is only a random relation between layers, then the overall "crystal" is actually composed of a set of two-dimensional crystals, and the diffraction effects proper to such crystals will be exhibited. The work of Warren[6] on carbon black is an example of this kind.

Similarly, some structures may be regarded as being composed of parallel rows of atoms with fixed relations between the rows. If, however, the relation between rows becomes random for some reason or other, then the material consists essentially of a set of parallel one-dimensional crystals. Many fibrous materials are of this type. Asbestos,[7] for example, shows streaks

[6] B. E. Warren, Phys. Rev. 59, 693 (1941).

[7] See, for example, Plate VI(a) (by Astbury) in K. Lonsdale, *Crystals and X-Rays* (G. Bell and Sons, London, 1948).

almost joining the layer line spots on a diffraction pattern, indicative of a partial loss of periodicity in a direction at right angles to the fibers.

Another way in which crystals of essentially only one or two dimensions can exist is as a second phase dispersed in a solid solution, for example, in the early stages of precipitation in certain age-hardening alloys. Here, two reciprocal lattices must be considered, that of the matrix and that of the precipitate, and there may or may not be certain relations between them. There is usually a definite orientation relationship but there is not necessarily a structural relationship, i.e., the reciprocal lattice rods due to a thin plate precipitate do not necessarily pass through the reciprocal lattice points due to the matrix. Stated in terms of the diffraction pattern, the streaks associated with the precipitate plate need not always connect with the diffraction spots from the matrix.

The first observations of essentially two-dimensional precipitation were made independently by Guinier[8] and by Preston[9] in 1938, using single crystals of Al-Cu alloys. By observations of streaks formed on diffraction patterns, they were able to show that preferential segregation of copper atoms occurred on some of the cube planes of the solid solution before actual three-dimensional precipitation occurred. The dimensions of these segregated regions (Guinier-Preston zones) were deduced from the dimensions of the diffraction streaks. More recently, Geisler and Hill[10] have found that the formation of planar Guinier-Preston zones is preceded, in the aging of Al-Ag and Al-Mg-Si alloys, by the formation of essentially one-dimensional precipitate regions.

The hexagonal close-packed structure of one form of cobalt is built up by stacking identical layers in a certain sequence. Faults may occur in the stacking sequence, interrupting the periodicity of the lattice, and when these faults are sufficiently close together, the material in between is essentially a two-dimensional crystal. Edwards and Lipson[11] discovered this effect in cobalt, both through the broadening of certain powder pattern lines and through the presence of streaks on single-crystal photographs.

The face-centered cubic structure is also built up by the stacking of identical layers in a certain sequence, and Barrett[12] has shown that stacking faults may be produced by the application of stress in the case of supersaturated Cu-Si alloys. The evidence here was the existence of streaks on oscillating crystal photographs.

It is to be anticipated that further examples of one- and two-dimensional crystals will be found in many materials as the methods of x-ray diffraction become more refined and the attention of investigators directed to the "minor" features of the diffraction pattern.

[8] A. Guinier, Compt. rend. **206**, 1641 (1938).
[9] G. D. Preston, Proc. Roy. Soc. (London) **A167**, 526 (1938).
[10] A. H. Geisler and J. K. Hill, Acta Cryst. **1**, 238 (1948).
[11] Olive S. Edwards and H. Lipson, Proc. Roy. Soc. (London) **A180**, 268 (1942).
[12] Charles S. Barrett, J. Metals, 123 (January, 1950).

I am writing you this letter . . . to tell you that we already have the short circuit machine and the coil, and we managed to obtain fields over 270,000 [gauss] in a cylindrical volume of a diameter of 1 cm, and 4½ cms high. We could not go further as the coil bursted with a great bang, which, no doubt, would amuse you very much, if you could hear it. The power in the circuit was about 13½ thousand kilowatts . . . but the result of the explosion was only the noise, as no apparatus has been damaged, except the coil. . . .

The accident was the most interesting of all the experiments, and gives the final touch of certainty, as we now know exactly what has happened when the coil bursted. We know just what an arc of 13,000 amperes is like. Apparently it is not at all harmful for the apparatus and the machine, and even for the experimenter if he is sufficiently far away.—P. KAPITZA, to Rutherford, (1925).

An Early Experimental Determination of Snell's Law

JOHN W. SHIRLEY
North Carolina State College, Raleigh, North Carolina
(Received November 14, 1950)

The law of refraction known by the name of Snell's law was circulated in manuscript by him as early as 1621. This paper shows that the law of sines was experimentally discovered independently in England by Thomas Hariot and his friends, and was used by them several years before 1621.

TO many scientists in the early seventeenth century, it appeared that the culmination of optical studies was reached with the enunciation of the law of sines governing refraction. Nearly every scientist in England and on the continent had been attempting to discover the mathematical rules governing refraction for half a century, both to enable them to work more accurately in designing lenses and telescopes and to determine some rules whereby parallax could be computed. As in many of the fields of mathematical science, it was Descartes who first formally enunciated the law—that the ratio of the sines of the angles of incidence and refraction is a constant—in its modern form, in his *Dioptrics*, published in Leyden in 1637. The word soon spread, however, that this was not an original contribution of Descartes, but that the law had been worked out geometrically by the brilliant young Dutch astronomer, Willebrord Snell (1591–1626) and had been circulated in manuscript by him as early as 1621. Consequently, the law of sines came to be known, as it is now known, as Snell's law.

Actually, the law of sines was discovered independently in England by Thomas Hariot and his friends, Walter Warner and Sir Thomas Aylesbury. Hariot began experimenting with refraction early in the 1590's corresponded with Kepler[1] on the subject from 1606 to 1609, and continued his experiments until about 1618, three years before his death.[2] Though the actual manuscript notes of these experiments do not appear to exist, an account of the way in which the experiment was conducted remains in the methodical notes of Dr. John Pell. Sometime about 1640, the date of Walter Warner's death, Pell gathered together his notes on Warner's conversations with him on mathematical matters. Among the British Museum manuscripts (Birch MSS 4407, fol. 183[a]) are Pell's neat notes and drawings on Warner's report. Modernized only in spelling and punctuation, these read as follows:

> 6. Mr. Warner says he had of Mr. Hariot this proportion
> As the sine of one angle of incidence
> to the sine of its refracted angle, found by experience
> So the sine of any angle of incidence upon the same superficies to the sine of its refracted angle, to be found by supputation.

but that he never saw him demonstrate it. "But," said he, "upon some occasion I did thoroughly demonstrate this proportion, but it was a long process."

I told him [that] Descartes in his *Dioptrics* (printed at Leyden, 1637) page 20, had the same proportion, *viz.:* As *AH* to *IG*, so *KM* to *LN* [Fig. 1], and that he demonstrates it that the refraction must be measured according to these lines, and not by the quantity of the angle either

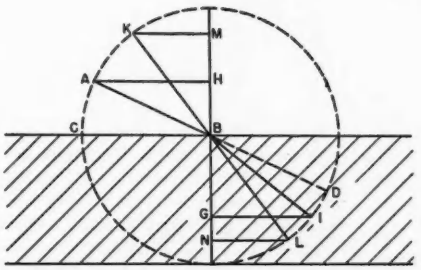

FIG. 1. Diagram to illustrate refraction. "Descartes··· had the same proportion, *viz:* As *AH* to *IG*, so *KM* to *LN*, and···demonstrates it that the refraction must be measured according to these lines, and not by the quantity of the angle either of incidence as *ABH*, or of the refract angle *GBI*, much less by the angle of refraction, as they call it, *IBD*."

[1] Kepler, in his *Prolegomena ad Vitellionem*, 1604, devoted much time searching for a law of refraction, but could find only a number of empirical rules on which to approximate the proper angle. Hariot's correspondence with Kepler may be found in letters 222–226, *Joannis Keppleri Aliorumque Epistolae Mutuae* (Frankfort, 1718).

[2] Hariot suffered from a cancer of the nose, and there is little evidence that he did much real study after 1618, and very little after 1616.

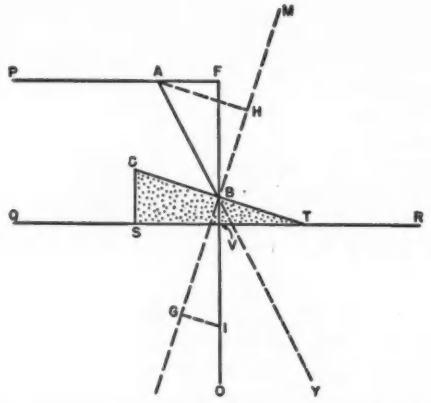

FIG. 2. Diagram to illustrate the method used by Hariot and his friends to measure the refraction of glass.

of incidence as *ABH*, or of the refract angle *GBI*, much less by the angle of refraction as they call it, *IBD*.

7. He told me that their manner of trial was thus: Upon a table about 2 yards long at Sir Thomas Aylesbury's house, they drew the line *FO* parallel to the side of the table, and *FP* at the end perpendicular to it. To this they drew a parallel *QR* through the middle of *FO*, crossing it at *V*. Upon this they laid *ST*, one of the legs of the polished triangle of glass, *CST*, and setting their eyes at *O*, another removed a pin (or the like bright object) in the line *FP* from *F* toward *P*, till they could see it through the glass. For thereby they knew [that] the ray of incidence fell upon *B*, the point where *CT* cuts *FO* (else they could not have seen it). The ray *BV* comes to the eye at *O* unbroken through the air, *VO*, because it is perpendicular to the superficies *ST*. Wherefore, making a mark at *A*, the place of the object, they measured *AF* and *BF* and so found the angle *ABF*, to which they added *FBM* (equal to *CTS*, the angle of the glass, measured before the beginning of the observation) and so knew *ABM* the angle of incidence (equal to *GBY*) and *ABO* is the radius.

$$GBO = FBM, \text{ the angle refracted.}$$
$$YBO = ABF, \text{ the angle of refraction.}$$

Thus he says they made trials with several triangles of glass and found the proportion of *AH* to *IG* (*AB*=*BI*) to be always, as it seems, as 10,000,000 to 6,528,703. For that number he set under the last angle refract, and that I suppose was his reason why he set a refract angle for 90, whereas it is certain that an angle of perpendicular incidence has no refraction.

[Dr. Pell proceeds to show that there must have been a slight error in the observation, since his computation shows the conclusion to be 9″ off.]

8. He said that they had the judgment of 3 or 4 pairs of eyes in this experiment, and that for a visible object they preferred a lamp with a board, held between it and the glass, having a straight edge.

It is not surprising that the English, the Dutch, and the French should all come to agreement on the laws of refraction: the study was a popular one and dozens of independent investigators must have been working on it simultaneously. It is interesting, however, that the Pell-Warner account of the experimental techniques employed by Hariot and his friends should be so modern. The equipment is simple; the methods employed are straightforward; and the results are verifiable both through repetition and the checking of several observers. Just as it stands, the experiment would serve for the use of physics students today, sure proof that experimental science was already firmly established during the years while Bacon was writing his philosophical *Novum Organum*.

Space and time, however, as human beings know them, are not in reality so impersonal as science pretends. Theologians conceive God as viewing both space and time from without, impartially, and with a uniform awareness of the whole; science tries to imitate this impartiality with some apparent success, but the success is in part illusory. Human beings differ from the theologians' God in the fact that their space and time have a here and now. *What is here and now is vivid, what is remote has a gradually increasing dimness. All our knowledge of events radiates from a space-time centre, which is the little region that we are occupying at the moment. "Here" is a vague term: in astronomical cosmology the Milky Way may count as "here," in the study of the solar system "here" is the earth, in geography it is the town or district in which we live, in physiological studies of sensation it is the brain as opposed to the rest of the body. Larger "heres" always contain smaller ones as parts; all "heres" contain the brain of the speaker, or part of it.*—BERTRAND RUSSELL, HUMAN KNOWLEDGE, 1948.

An Improved Mechanical Equivalent of Heat Experiment

GEORGE G. KRETSCHMAR*
Walla Walla College, College Place, Washington
(Received March 30, 1951)

An interesting variation of Joule's classical experiment on the mechanical equivalent of heat is described. It makes use of a rotary stirring mechanism for adding energy to water in a calorimeter, the energy being measured by a bifilar suspension of the driving motor. The motor is operated at synchronous speed. Temperature measurements are by means of a thermocouple and potentiometer. With reasonable care results can be obtained which are consistent to about 3 percent.

THE concept of the mechanical equivalent of heat is so fundamental that some kind of experiment for the verification of the numerical constant is included in most courses in experimental physics. The indirect method, in which energy is put into a calorimeter by means of an electrical heating coil, and is measured with electrical measuring instruments, is easy to carry out, and with reasonable care and simple equipment, the results are satisfactory. The same cannot be said, however, of some of the direct experiments in which the energy is put into the calorimeter by means of stirring, or with a friction device, and the energy measurements are in terms of mass, length, and time.

With the energy input by means of a stirring mechanism, as in the original experiments of Joule, the temperature difference, for usual types of simple equipment, is too small for satisfactory precision. This is a seriously limiting factor in verifying Joule's constant. Joule himself discovered this limitation,[1] as we gather from some of the descriptions of his original presentation of the conservation of energy principle, before the British Association at Oxford in 1847.

A very satisfactory experiment, using a stirring device, can, however, be carried out by making use of a bifilar suspension[2] for measuring the work input. By making use of a small synchronous motor[3] for driving the stirrer, the motor speed is known to high precision, and need not be measured by the student.

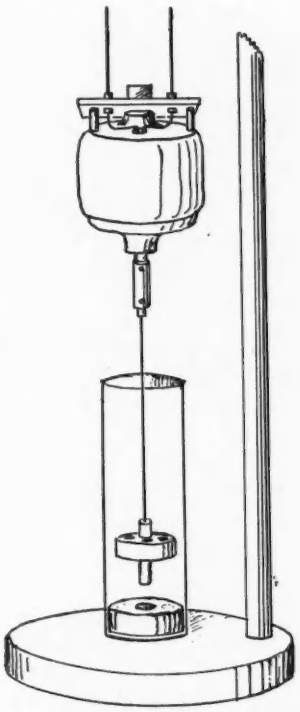

FIG. 1. Apparatus for mechanical equivalent of heat experiment.

* Now at Michelson Laboratory, Inyokern, California.

[1] "Miller or Graham or both were for many years quite incredulous of Joule's results, because they all depended on fractions of a degree of temperature···I remember distinctly at the Royal Society, I think it was either Graham or Miller saying simply he did not believe Joule, because he had nothing but hundredths of a degree to prove his case by."—From a letter to J. T. Bottomley written by William Thompson about 1882. See E. C. Watson, Am. J. Phys. 15, 384 (1947).

[2] Sutton, *Demonstration Experiments in Physics* (McGraw-Hill Book Company, Inc., New York, 1938), p. 245.

[3] Such a motor may be easily made from a small single phase 4-pole induction motor having a "squirrel cage" rotor by making four saw cuts radially into the rotor, thus isolating four equal conducting sections. After this operation it will run synchronously, but with reduced power output. Small fractional horsepower synchronous motors may be purchased from Bodine Electric Company, Chicago, Illinois.

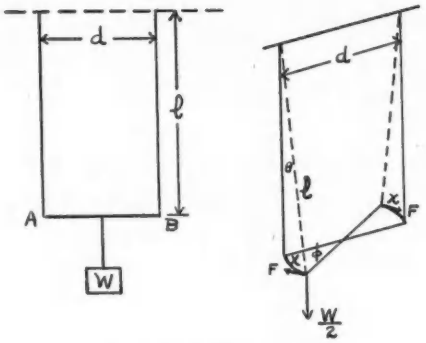

FIG. 2. The bifilar suspension.

The apparatus, shown in Fig. 1, consists of a rigid support which holds up the motor by means of the two wires which carry the driving current. Hanging from the vertical shaft of the motor is the stirring device, which is a thick brass disk smaller in diameter than the calorimeter cup. The disk is supported by a steel piano wire 0.35 mm in diameter, and it has four quarter-inch holes drilled symmetrically and at an angle which produces a slight turbine effect and gives better stirring action to the water.

For stabilizing the rotation, a quarter-inch brass rod projects down from the bottom of the disk and fits rather loosely into a hole in an aluminum frame, which serves as a lower bearing, as shown in the figure. A front-surfaced mirror, mounted on top of the motor, is used in connection with a telescope and scale for finding the angle of twist produced by the reaction while the experiment is in operation. The tall aluminum cup is the core of a vacuum-type calorimeter.[4] Temperature measurements are made by means of a copper-constantan thermocouple used in connection with a Leeds and Northrup type K2 potentiometer.

With the aid of Fig. 2 the simple theory may be worked out. The rigid bar AB is twisted through a small angle ϕ. For half the weight W concentrated at each end, the restoring force F is given by

$$F=(W/2)\sin\theta=(W/2)(x/l).$$

Then the torque, for small angle of twist,

$$Fd=(W/2l)(d\,\phi/2)d$$
$$=Wd^2\phi/4l.$$

The work done

$$=\text{torque}\times\text{angular distance}$$
$$=(Wd^2\phi/4l)(2\pi nt),$$

where $n=$number of revolutions per minute and $t=$the time in minutes.

The temperature difference should be measured to a precision of about 0.01 C°. This requires that the water in the calorimeter be in thermal equilibrium with the outside case of the calorimeter at the beginning of the experiment, so that it is best to fill the calorimeter with

TABLE 1. Results of two 10-minute trials.

Weight of aluminum (cup and bearing)		84.0 g
Weight of motor and stirrer		6440.0 g
Weight of stirrer (immersed part)		124.0 g
Length of bifilar suspension		57.1 cm
Distance between wires		8.70 cm
Speed of synchronous motor		1800.00 r.p.m.
	Trial 1	Trial 2
Time of run	20 min	20 min
Distance of scale	376 cm	376 cm
Average deflection	23.9 cm	24.3 cm
Potentiometer reading—start	0.740 mv	0.710 mv
Potentiometer reading—finish	0.778 mv	0.743 mv
Equivalent temperature difference	0.95 C°	0.825 C°
Weight of aluminum container and water	449.3 g	487.0 g
Total water equivalent	393.0 g	430.7 g
Torque $(Wd^2\phi)/(4l)$	67.82 g cm	68.96 g cm
Work $=2\pi nt\times$torque	15.34×10^6 g cm	15.60×10^6 g cm
$J=980W/H$	4.026×10^7 ergs/cal	4.302×10^7 ergs/cal
Mean value of trials 1 and 2	4.16×10^7 ergs/cal	
Percent difference from mean	3.3 percent	3.3 percent

[4] The one we use is manufactured by Central Scientific Company, Chicago, Illinois, Catalog No. 78044.

water the day before the experiment is to be run. Final temperature measurements are made only after complete mixing of calorimeter contents. Since the actual rise in temperature is less than one degree Centigrade it is not necessary to make a radiation correction.

The experiment gives rather consistent results which are fairly close to accepted values for Joule's constant. Table I, which gives the data and computed results of two twenty-minute runs taken by the author, shows about what may be expected from the experiment.

The Moment of Inertia and the Period of the Rocking of an Elliptic Band

JOHN SATTERLY
University of Toronto, Toronto, Canada
(Received March 26, 1951)

The radius of gyration of a thin, elliptical band of uniform line density is given (*i*) by direct evaluation, (*ii*) from the point of view that the band represents the interval between adjacent ellipses of semiaxes a and b, and (*iii*) by taking the band between two confocal ellipses. Values of the radii of gyration for several elliptic bands are computed from the three points of view. Experiments in which elliptic bands of brass and glass were rocked on glass plates and on suspended knife-edges gave agreement within 5 percent of the computed radii of gyration.

IN the October, 1950 issue of this journal[1] I gave a short description of the rocking of bodies to-and-fro on a horizontal table. The period of vibration (for small amplitudes) of a uniform elliptic plate was given in the form

$$T=2\pi\{(k^2+h^2)/(R-h)g\}^{\frac{1}{2}},$$

where the letters are as defined in the paper. For the elliptic plate $k^2=(a^2+b^2)/4$ and $R=a^2/b$, hence L, the length of the equivalent simple pendulum, is given by

$$L=\tfrac{1}{4}b(a^2+5b^2)/(a^2-b^2).$$

Since $T=2\pi(L/g)^{\frac{1}{2}}$ and g in Toronto $=980.43$ cm sec^{-2} we find

$$T=0.2007L^{\frac{1}{2}}$$

or very nearly $T=\frac{1}{5}L^{\frac{1}{2}}$.

I now add the *elliptical band* to my former list of rocking bodies.

CALCULATIONS OF ITS MOMENT OF INERTIA

(1) The value of the moment of inertia of a thin elliptical band of uniform line density does not appear in any of the tables that I have consulted. The deduction of its value is not easy, but Professor Colin Barnes of the Department of Physics has worked it out for me. If M is the mass of the band and a and b are the mean semiaxes of the ellipse the moment of inertia of the band about an axis through the center of the ellipse at right angles to its plane is given by

$$I=\frac{M}{3}\left(a^2+b^2+b^2\frac{F_1}{E_1}\right)$$

and, therefore, the square of the radius of gyration is given by

$$k^2=\frac{1}{3}\left(a^2+b^2+b^2\frac{F_1}{E_1}\right), \qquad (1)$$

where F_1 and E_1 are the complete elliptic integrals of the first and second class, respectively, the parameter θ of the tables[2] being given by $\sin\theta=\{(a^2-b^2)/a^2\}^{\frac{1}{2}}=$ the eccentricity of the ellipse. Because of its length I omit the proof of Eq. (1). Professor Barnes' proof included also the values of the moments of inertia about the two principal axes.

The perimeter of the ellipse is given, of course, by $4aE_1$, or by the approximate expressions containing the eccentricity given in the textbooks,

[1] J. Satterly, Am. J. Phys. **18**, 405 (1950).

[2] I use J. B. Dale's *Five-Figure Tables of Mathematical Functions* (Edward Arnold, London, England).

or even by the still more approximate expression $\pi(a+b)$.

(2a) For a thin elliptical band which is really the interval between an ellipse of semiaxes a and b and a larger ellipse of semiaxes $a+w$, $b+w$, where w is small compared with a and b, we may use the expression

k^2 = (difference between the second moments of the two elliptic areas)/(difference between the elliptic areas). (2)

This may be written

$$k^2=\frac{\pi(a+w)(b+w)[(a+w)^2+(b+w)^2]-\pi ab(a^2+b^2)}{4\pi(a+w)(b+w)-4\pi ab}$$

and reduces to $k^2=\{(a+b)/2\}^2$, if we neglect squares and higher powers of w. Inclusion of the next term multiplies this value of k^2 by $(1+2w/(a+b))$.

(2b) Employing the calculus the result may be obtained as follows: Write the second moment as

$$I=\pi ab\{(a^2+b^2)/4\}=\tfrac{1}{4}\pi(a^3b+ab^3)$$

and the area as $A=\pi ab$, and we get

$$\Delta I=\frac{\partial I}{\partial a}\Delta a+\frac{\partial I}{\partial b}\Delta b$$

$$=\tfrac{1}{4}\pi\{(3a^2b+b^3)\Delta a+(a^3+3ab^2)\Delta b\},$$

which, if $\Delta a=\Delta b$, becomes

$$\Delta I=\tfrac{1}{4}\pi(a+b)^3\Delta a.$$

Also in the same way $\Delta A=\pi(a+b)\Delta a$, and therefore

$$k^2=\Delta I/\Delta A=\{(a+b)/2\}^2 \qquad (3)$$

as before. Note that this thin band will not have a uniform line density.

(3) A third approach is to take the thin band between two *confocal* ellipses. If the standard ellipse has semiaxes a and b and eccentricity e and the near-by confocal ellipse has semiaxes a' and b' and eccentricity $e'=e+de$ then, since ae must equal $a'e'$ we have, neglecting squares of de, $a'=a(1-de/e)$,

$$b'=a'(1-(e+de)^2)=b\{(1-de)/(e(1-e^2))\}.$$

The second ellipse is within the standard ellipse. The area between the two ellipses

$$=\pi ab-\pi a'b'=\pi ab\{(2-e^2)/(e(1-e^2))\}de.$$

The second moment of the area between the two ellipses

$$=\tfrac{1}{4}\pi ab(a^2+b^2)-\tfrac{1}{4}\pi a'b'(a'^2+b'^2)$$

and works out to be

$$\pi ab\{a^2(4-3e^2)+b^2(4-e^2)\}de/(4e(1-e^2)),$$

whence

$$k^2=\tfrac{1}{4}\{a^2(4-3e^2)+b^2(4-e^2)\}/(2-e^2). \qquad (4)$$

If, in this expression, we replace e^2 by its value in terms of a and b, *viz.*, $e^2=(a^2-b^2)/a^2$, we find

$$k^2=\tfrac{1}{4}(a^4+6a^2b^2+b^4)/(a^2+b^2), \qquad (5)$$

which is a simpler expression to use as a and b are the measured quantities.

Note that with the confocal ellipses the band has not the same width at the ends of the major and minor axes and is therefore not of uniform line density.

For elliptic bands of the three types above and having $a=5.000$ and $b=3.000$ cm, respectively, Eqs. (1), (3), and (5) give $k^2=16.02$, 16.0; and 15.12 cm², respectively. The low value of k^2 with the confocal band is the result of the width at the ends of the minor axis being greater than the width at the ends of the major axis. Table I following shows how the results of Eqs. (1), (3), and (5) compare as the shape of the

TABLE I. The three values of the k^2 of elliptic bands are calculated from Eqs. (1), (3), and (5), respectively.

a	12.00	15.00	20.00	30.00	40.00	11.65
b	10.00	10.00	10.00	10.00	10.00	8.50
k^2	0.3056	0.5565	0.7500	0.8889	0.9375	0.4669
k	0.5527	0.7454	0.8660	0.9428	0.9680	0.6833
θ	33.55°	48.20°	60.00°	70.50°	75.50°	43.10°
F_1	1.716	1.904	2.156	2.527	2.800	1.827
E_1	1.442	1.321	1.211	1.113	1.072	1.367
b^2	100.0	100.0	100.0	100.0	100.0	72.2
$b^2\frac{F_1}{E_1}$	119.7	144.2	178.1	227.0	261.2	96.6
a^2	144.0	225.0	400.0	900.0	1600.0	135.7
The sum	363.7	469.2	678.1	1227.0	1961.2	304.5
k^2 (Eq. (1))	121.2	156.4	226.0	409.0	653.7	101.5
k^2 (Eq. (3))	121.0	156.0	225.0	400.0	625.0	101.5
k^2 (Eq. (5))	120.0	150.5	205.0	340.0	519.1	99.1

ellipse is varied. Except in the last column b is kept constant at 10 cm and a varied. I omit the linear units in the table.

The first two values of k^2 agree fairly well except when the ellipse becomes long and thin, so that for the ellipse that is usually drawn or made the simple expression of Eq. (3) is sufficiently precise.

The period of rocking of an elliptic band

$$=2\pi\{(k^2+b^2)/R-b\}^{\frac{1}{2}}.$$

Substituting for R its value of a^2/b we find for the length L of the equivalent simple pendulum

$$L=b(k^2+b^2)/(a^2-b^2).$$

EXPERIMENT

The workshop made an elliptic band of uniform linear density by cutting off a ring about one inch wide from a large circular brass pipe of about 20 cm diameter and 2 mm wall thickness. This band was squeezed into nearly elliptical shape, using for comparison previously drawn ellipses and finally fitted around a wooden elliptic disk that had been made by trial and error to have the correct inside perimeter (63.2 cm).

Dimensions of Band

Inside $2a=23.1$ cm,	outside $2a=23.5$ cm.
Inside $2b=16.8$ cm,	outside $2b=17.2$ cm.
Mean $a=11.65$ cm,	mean $b=8.50$ cm.

The steps in the calculation of its k^2 are given in the last column of the Table I above.

Period of Rocking on a Horizontal Glass Plate

Resting on the B end of the minor axis $T=1.00$ sec.
Resting on the B' end of the minor axis $T=1.12$ sec.

Ten vibrations were timed for each determination of T, so that the amplitude was fairly large at first and small at the end of the run.

CALCULATED PERIOD

$$L=8.50(101.5+72.2)/(135.2-72.2)$$
$$=23.48 \text{ cm};$$

whence

$$T=\pi(L)^{\frac{1}{2}}=0.97 \text{ sec.}$$

The agreement between the observed and calculated values of T is fairly good for the B end. The B' end must have had a slightly different curvature.

As a check on the value of k^2 the band was suspended from an upturned fixed knife edge and swung as a compound pendulum. Fifty swings were timed in each case. Curiously enough the period for this particular ellipse was found to be 0.91 sec whether hung from the ends of the major or minor axes or from any other point where the band could be got to rest. The length of the simple equivalent pendulum $=L'=20.56$ cm[3] and putting $20.56=a+k^2/a$, or $=b+k^2/b$ we find k^2 to be 103.8 and 102.5 cm^2, respectively.

This was confirmed also with a torsion pendulum.

A larger elliptic band of dimensions $2a=37.9$ cm, $2b=25.0$ cm, $w=2$mm gave periods of rocking about B and B^1 of 1.04 and 0.98 cm, respectively. Calculation gave $k^2=248$ cm^2 whence $T=1.00$ sec.

Another elliptic band was obtained by cutting a band from an elliptic-sectioned large medicine bottle. Theory and experiment gave fairly good agreement.

I am now on the lookout for other elliptical bands and think I ought to find some elliptical napkin-rings in the big multiple stores, or some elliptical billiard balls as sung of by Gilbert and Sullivan.

[3] I use here $T=2\pi(L/g)^{\frac{1}{2}}$, and as g in Toronto $=980.5$ cm/sec^2, $T=0.2007L^{\frac{1}{2}}$ or $L=24.83T^2$.

The ordinary crude mind has only two compartments, one for truth and one for error; indeed the contents of the two compartments are sadly mixed in most cases; the ideal scientific mind, however, has an infinite number. Each theory or law is in its proper compartment indicating the probability of its truth.—HENRY A. ROWLAND.

Photometric Teaching Methods Using Photoelectric Cells

U. Andrewes and T. J. Dillon
King's College (Household and Social Science), University of London, London, England
(Received February 14, 1951)

Construction details of three photometers, designated as black box, integrating, and hoop types, are given. All photometers use photovoltaic cells as the light-measuring devices. Tests made with these photometers upon filament-type lamps and fluorescent discharge tubes establish the mean horizontal candle power and the mean spherical candle power of light sources normally found in the laboratory. Auxilliary experiments are described for determining the range of validity of the inverse square and cosine laws for the light sources. A table lists the luminous flux for several sources, measured by each type of photometer.

ALTHOUGH the study of photometry is becoming of increasing importance in many branches of physics, laboratory experiments in this branch of optics have not been developed to the same extent as those in other parts of the subject. These experiments using photoelectric cells are suggested as being suitable for college courses. They comprise simple methods of carrying out photometric measurements using apparatus that can be constructed in the laboratory workshop and the type of light source normally found in a laboratory. The experiments were done using lamps of widely different types; gas-filled metal filament lamps, metal filament vacuum lamps, carbon filament lamps, and fluorescent discharge tubes.

It was found that the photoelectric cells which best conform to the requirements of these photometer experiments are of the photovoltaic type. Their sensitivity is high, they are stable and permanent and no external battery is necessary for their operation. A further advantage is that a linear relationship holds between illumination and current output, if the external resistance is small compared with the internal resistance of the cell when a current is flowing.

For these experiments microammeters and galvanometers of suitable low resistance (e.g., less than 150 ohms) were used, and it could thus be assumed that the illumination response of the cells was linear.

The calibration constants of the cells, under fixed conditions of external resistance, were determined using a regulation photometer bench. The standard source was a specially constructed tungsten vacuum lamp with uniplane filament. This had been previously calibrated and the mean horizontal candle power of the lamp was accurately known.

Preliminary experiments, using a vacuum emission photocell, in which saturation current is directly proportional to illumination, showed that the inverse square law held for distances from the lamp greater than 29 in. The photovoltaic cells were all calibrated by taking current readings for distances greater than 29 in. between lamp and cell, calculating the illumination in foot candles and plotting graphs between current and illumination. It will be appreciated that, as the difference of color in the light output of the lamps is considerable, a lamp of one type could not easily be compared with a lamp of another type using a photoelectric cell method, unless the spectral response curve of the cell is similar to the visibility curve of the eye. Most makers of cells claim that the spectral response of the cells approximate to this curve, but, if the color of a source differs considerably from that of the standard source, the cell can be provided with a filter by means of which the cell response is practically similar to that of the eye. The filters supplied by the makers are either detachable or incorporated in the cell mounting.

Experiments described in this paper were carried out with both filtered and unfiltered cells, and, it was found that, except in the case of a daylight fluorescent lamp, the use of filters was not essential for the purposes for which the work was intended.

As the light characteristics of lamps are affected by changes of operating temperature, it

is necessary to use some method of voltage control to compensate for fluctuations in the mains voltage supply. This was accomplished by a hand-controlled series resistance, by a potentiometer arrangement, or by the use of a constant voltage transformer.

In order that experiments may be performed anywhere in the laboratory and not in a special dark room, three experimental units were devised and constructed in the laboratory: (*i*) a black box photometer for candle power measurement in one direction, (*ii*) a cubical integrating photometer for measurement of total flux, (*iii*) a hoop photometer for the determination of polar curves of light distribution.

APPARATUS AND EXPERIMENTAL METHODS

I. The Black Box Photometer

The photometer is shown in Fig. 1. It is made of wood, approximately 40×10×12 in., one long side (the front) being hinged at its lower edge. The inside is painted dull black and the walls immediately surrounding the lamp L are covered with black velvet. The base of the box consists of an optical bench with carriers for the lamp and photoelectric cell P. Three wooden screens S with circular holes 3.5 in. in diameter fit through slots in the top of the box and are arranged between the lamp and cell so that only direct light from the lamp can reach the cell. The lamp filament, screen holes, and cell are leveled, the face of the photocell set at right angles to the bench axis and the box is closed during observations. Experiments which can be carried out with this photometer are described briefly.

(*a*) *Comparison or measurement of mean horizontal candle powers of incandescent filament lamps.*—The comparison of the mean horizontal candle powers (m.h.c.p.) of two lamp sources can be carried out using this photometer by placing the photocell in turn at a fixed distance from the source and by comparing the photocell currents.

If I is the m.h.c.p. of lamp, $I=Kir^2$, where K is the calibration constant of the cell, i the current and r is the distance from lamp to cell. For accurate results, readings of the current are taken at several distances for each source and

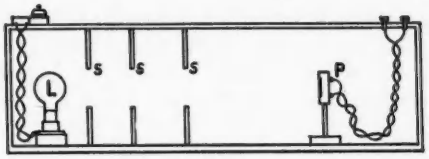

FIG. 1. Vertical section of black box photometer.

graphs of i and $1/r^2$ are plotted. These are found to be straight lines and the slopes of the lines are used in the calculation. If a calibrated cell is used, absolute determination of m.h.c.p. may be made.

Owing to the asymmetry of the lamp filaments, it is necessary with most types of lamp to rotate the lamp about its axis of symmetry and to take several readings at each distance, the number of readings required depending on the type of lamp filament.

(*b*) *Determination of brightness of a fluorescent tube.*—A hole was made in the back of the lamp compartment of the black box and one end of a fluorescent tube passed through it. The tube is arranged at a suitable height with its axis horizontal and at right angles to that of the optical bench. In order to limit the size of the source, two cylinders of black paper can be fitted over the tube and adjusted to leave a length of surface exposed between their edges. In this case a filtered cell must be used.

If I is the luminous intensity per unit length, $2a$ the diameter of the tube, B the brightness in candles per unit area, E the illumination produced at distance r, and l the length exposed, then $E=Il/r^2$ and $I=2aB$. By measuring the photocell current i for various values of l and r, a graph of i and l/r^2 may be plotted, from the slope of which, knowing the calibration constant of the cell, I and hence B can be determined.

(*c*) *Measurement of transmission factor of glass or of a transparent liquid.*—The box is placed with its long axis vertical and with the lamp compartment at the top. A monochromatic source of light is used. The photocell lies face upwards on the bottom of the box and a slab of transparent solid, or a liquid contained in a vessel with a transparent base, is placed directly on the face of the cell. The screens are arranged so that there is no reflection from the walls of the box or of the vessel which contains the liquid.

Readings of the photocell current are taken for different thicknesses of absorbing medium. For any thickness t, T_λ, the transmission factor for wavelength λ is given by the ratio i_1/i_2 where i_1 is the current with the medium present and i_2 the current with no absorbing medium.

It can be shown theoretically that $\log(1/T_\lambda)$ is proportional to t, where $1/T_\lambda$ is the optical density. This can be verified experimentally from the results if the necessary corrections for refraction and for reflection at the first surface of the medium are made.

Water is not a suitable medium for these experiments because, for a thin layer, the visible region is transmitted without absorption or T_λ is approximately unity.

(*d*) *Investigation of the effects of voltage variation on lamp characteristics.*—A variable transformer is used to alter the supply voltage, and, using the photocell at a fixed distance from the lamp, the illumination is measured for a range of voltages. If the variation is not greater than 15 percent above or below the rated value it is unnecessary to use a filtered cell to allow for color changes.

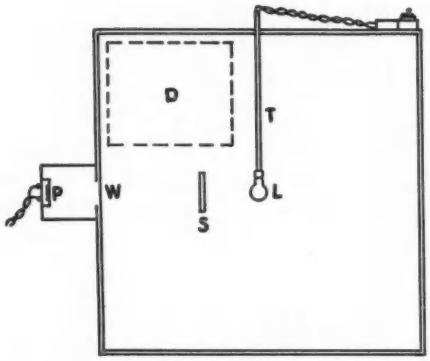

FIG. 2. Vertical section of integrating photometer.

Results are found to be in accordance with the empirical formula $\log F/F_0 = B \log V/V_0$, where F_0 and V_0 are luminous output and voltage under standard working conditions, F and V the corresponding quantities under any other conditions, and B is a constant depending on the type of lamp.

(*e*) *Investigation of the cosine law.*—One of the disadvantages of the early photovoltaic cells was that they did not follow the cosine law closely, and therefore were unsuitable for the measurement of general illumination. The black box may be used to determine how far any particular cell deviates from the law, by mounting the cell so that it can be rotated through known angles, the illumination produced by a lamp being measured at each angle.

It was found that modern cells showed quite good agreement with the cosine law provided the angle of incidence does not exceed 40°, and, for some cells, the deviation was not appreciable up to angles of 65° or 70°.

II. Integrating Photometer

For the comparison of total luminous flux, the ideal piece of apparatus is an integrating photometer in the form of a hollow sphere. The difficulty of construction of such a sphere is considerable and an integrator of cubical form is quite suitable for comparing lamps of the same type. The sphere, or cube, must be large compared with the size of the source and, in order to be able to measure the flux of many types of lamps, including small fluorescent tubes, the cube was made with side one meter in length. It is of aluminum on a wooden framework and a vertical section is shown in Fig. 2.

Incandescent filament lamps are supported by a lamp holder attached to a metal tube T which passes through the center of the top of the cube. Small fluorescent lamps can be slung horizontally by fine connecting wires. In the center of one vertical face is the rectangular window W (2.75 in. square) covered by a sheet of Ilford diffusing medium, its surface as nearly as possible flush with the inner surface of the cube. Outside the window the photocell P is housed in a small box with the interior painted dull black. In one side of the cube a door D gives access to the interior and at a third of the distance betwen lamp and window a small screen S is suspended. The interior of the cube, including tube, lamp holder and screen is painted mat white.

To compare the total flux from two sources, readings are taken of the photocell current when the lamps are placed in turn in the holder. If a standard lamp of known flux is used, that of any other lamp of the same type can be calcu-

lated directly. Even with lamps of different polar distributions the error introduced by using a standard of different type is not great.

The lamps used as flux standards were of the gas-filled general service type standardized to a commercial accuracy.

III. The Hoop Photometer

An iron hoop of diameter about 4 ft is fixed with its plane vertical in a square wooden frame. (Fig. 3.) The lamp is arranged at the center of the hoop at the lower end of a metal tube T, which can be rotated about a vertical axis. A small fluorescent tube can be hung centrally by the leads, with the axis along a diameter. The circumference of the hoop is calibrated in degrees. The whole apparatus is surrounded by black curtains, and the wood and metal work are painted black. The photocell P can be moved round the circumference with the sensitive surface of the cell kept at right angles to the line joining the cell to the center of the lamp. Typical experiments which can be carried out using the photometer are described.

(*a*) *The polar curve of light distribution.*—The polar curve in any one vertical plane is obtained by taking photocell current readings as the cell is moved round the hoop. As in the case of the black box it is necessary, for accurate results, to rotate the lamp about its axis of symmetry, and to take a mean value of the photocell current for the different positions of the lamp. As the current is proportional to the illumination at the cell surface, the required polar curve is obtained by making the length of the radius vector proportional to the current for each angular position of the cell. If the calibration constant of the cell is known, the mean spherical candle power, m.s.c.p., of the source can be calculated from the curve using Russell angles.[1] Hence the flux F in lumens (m.s.c.p.$\times 4\pi$) can be determined.

Examples of polar curves obtained are shown in Fig. 4.

(*b*) *Spherical reduction factor.*—The ratio of the m.s.c.p. to the m.h.c.p. of a source is known as the spherical reduction factor. For a point source of light this factor is necessarily unity. When a source is a simple shape, the factor can be calculated, e.g., for a cylindrical source it is $\frac{1}{4}\pi$ or 0.785. In the case of metal filament, gas-filled lamps the intensity varies with the direction from the source as shown by the polar curves. As the weighted mean of the currents found using Russell angles is proportional to the m.s.c.p., and the mean value of the current readings in the horizontal plane is proportional to the m.h.c.p., the ratio of two means gives the reduction factor by a method which is independent of the calibration constant of the cell.

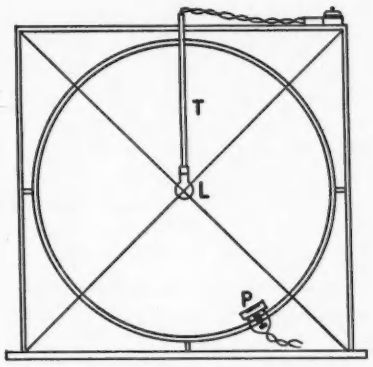

FIG. 3. Vertical section of hoop photometer.

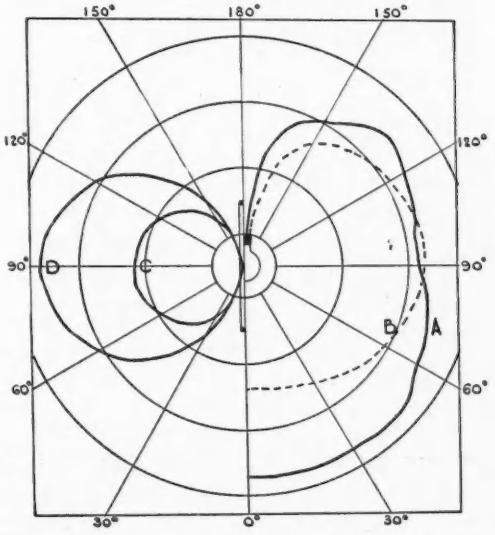

FIG. 4. Polar curves of gas-filled pearl lamp (A), carbon filament lamp (B), vacuum tungsten filament lamp (C), and small fluorescent tube (D).

[1] For a definition of these see *Dictionary of Applied Physics*, Glazebrook, ed. (Macmillan, London, 1923), Vol. 4, p. 433.

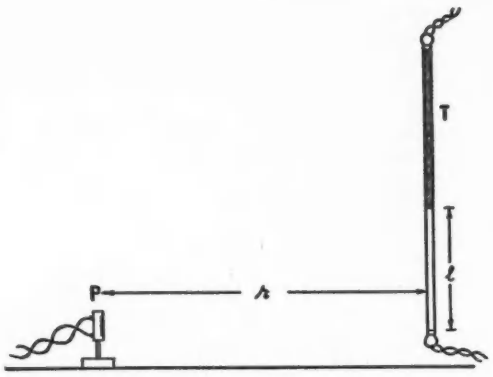

FIG. 5. Arrangement of apparatus for measuring illumination from an extended source.

The experimental values obtained by this method are included in Table I. For gas-filled lamps, the factors were found to be less than unity for the smaller power lamps and greater than unity for those over 40 w. A vacuum squirrel cage filament lamp or a small daylight fluorescent tube can be regarded as a cylindrical source. The results obtained for these sources agreed with theory.

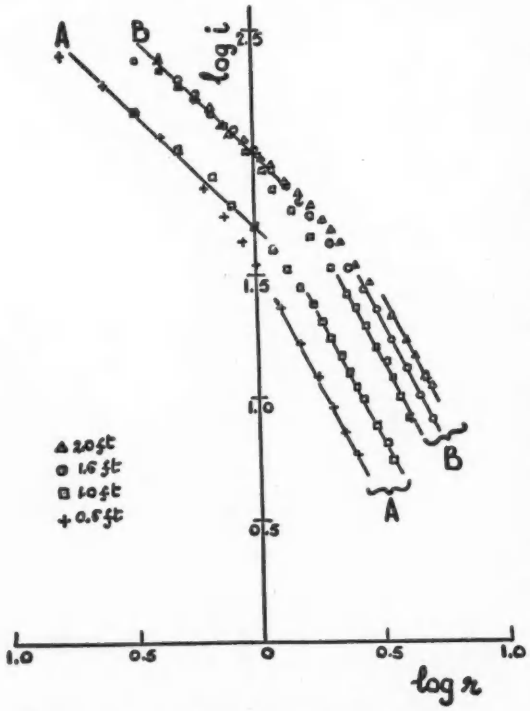

FIG. 6. Graphs of log$i$ and log$r$ illustrating the inverse and inverse square laws for fluorescent tubes 15 watt (A) and 80 watt (B) with different lengths exposed.

LIMIT OF RANGE OF VALIDITY FOR INVERSE SQUARE LAW

When the source is of finite dimensions, as in the case of a filament lamp, the inverse square law can be applied only where the distance from the source is large compared with the dimensions of the lamp. In the experiments using the black box photometer, it was found that, provided the diameters of the lamp bulbs were not more than about 2.5 in., satisfactory results were obtained for values of r greater than 16 in.

For a line source, the illumination at any point on a plane through one end at right angles to the line can be shown theoretically to be given by $E=(I/2r)[\tan^{-1}l/r+lr/(l^2+r^2)]$, where I is the intensity of unit length, r the distance of the point from the line and l the length of the line. It is evident that for values of r small compared with l the illumination is inversely proportional to the distance but for large values of r the inverse square law will hold. A long fluorescent tube can be regarded as a line source and measurements of the illumination produced by it at any point can be used to verify these relationships.

TABLE I. Photometric results.

Lamp (200–240 v)	Lamp rating (watts)	m.h.c.p. black box	m.s.c.p. hoop	Reduction factor hoop	Flux (lumens) black box	Flux (lumens) hoop	Flux (lumens) integrator
gas-filled pearl	15	9.85	8.6	0.91	114	108	109
	15	12.2	9.7	0.93	136	121	124
	25	21.7	20.1	0.90	244	253	238
	25	22.5	20.0	0.88	248	252	242
	40	27.1	30.0	1.11	377	377	364
	40	30.9	32.9	1.12	436	414	392
	60	49.4	53.6	1.10	685	674	655
	60	48.3	51.4	1.10	668	646	635
carbon filament	98	28.6	25.5	0.89	323	322	334
tungsten filament vacuum	24	17.8	13.8	0.80	179	174	184
fluorescent	15	57.0	45.0	0.78	590	563	590
daylight	80	410.0	—	—	4050	—	—

The tube T was supported vertically (Fig. 5) in a dark enclosure with the photocell P vertical and facing the tube, and the photocell current was measured for various values of l and r. In Fig. 6 log$i$ is plotted against log$r$ for two tubes of different size and the ranges for which the in-

verse law and the inverse square law hold can be seen from the slopes of the lines.

SUMMARY OF PHOTOMETRIC RESULTS

Typical results for twelve lamps of four different types are given in Table I. In the last three columns values of the total flux for each lamp are given, with the exception of the 80-watt fluorescent daylight lamp which was too large to be used in the hoop and integrator. The "black box values" have been calculated from the m.h.c.p. combined with the reduction factor to give the m.s.c.p. and hence the total flux. The "hoop values" have been derived from the m.s.c.p. and the "integrator values" found by direct comparison with a standard lamp.

These experiments were carried out at intervals during the past year, and in practice, the characteristics of a lamp alter with hours of running throughout its life. If the three experiments for any particular lamp had been performed on any one day, the results would probably show better agreement than those given in Table I. It is felt however that the results obtained amply justify the suggestions that these methods could profitably be introduced into University courses.

The calibrations of the standard lamps used in these experiments were carried out by the Research Laboratories of The General Electric Company Ltd., of England, to whom our thanks are due. We also thank Miss M. M. H. Henderson, and Miss V. Jackson, who carried out much of the experimental work, and Miss F. M. Spaull for assistance in construction of the apparatus.

The Knotty Masspoint *versus* the Rigid Tinplate

DANIEL A. NAYMIK
Michigan State Normal College, Ypsilanti, Michigan
(Received April 11, 1951)

There is an experiment in sophomore physics called "The Force Board Experiment" or "The Composition and Resolution of Nonparallel, Concurrent Forces." In the opinion of the author of this article, this experiment does not adequately bridge the gap between the mathematical process of vector addition and the "Study of Forces acting on a Crane," which is another classical experiment. For this reason the author has originated an experiment which is slightly more difficult than "The Force Board Experiment," and yet not as difficult as the crane experiment; it is called "The Tinplate Experiment."

THE TINPLATE *VERSUS* FORCE BOARD

THOSE teachers who appreciate the sense of satisfaction derived from presenting a topic by means of a logical chain of reasoning that is *logical* to the student also, will find that the tinplate experiment is far superior to the force board. The advantages of this new experiment will become quite apparent with its use in the classroom or laboratory.

The tinplate gives to the student four very important facts. (1) It proves to him that vector mathematics *must* be used as the algebra of forces. (2) It provides him with a real physical interpretation of a vector equation as applied to forces (see Eq. (1) and (2)). (3) It provides him with one necessary condition for equilibrium of a rigid body. (4) It introduces him to the three characteristic properties of forces. The instructor, therefore, has a place to begin a logical chain of lectures based on these four facts.

In addition, the tinplate serves as an introduction to the study of force systems. The instructor can then feel reasonably certain that the study of a system of parallel forces on a rigid body, and of a system of nonparallel, nonconcurrent forces on a rigid body will be a successful one from the student's point of view.

There is no doubt, on the other hand, that the old force board experiment has its advantages also, but the results obtained from this experiment are not so elegant and appealing as those of the tinplate. The force board experiment simu-

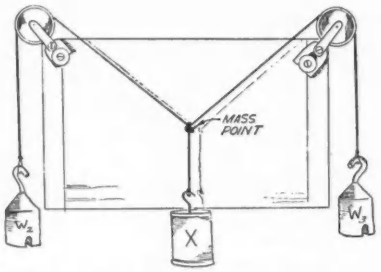

FIG. 1. The force board. Here W_2 and W_3 are known weights; X is a loaded tin can of unknown weight. The mass point is the knot which forms the intersection of the three supporting strings.

lates the study of three forces acting on a *mass point*. A point mass is a concept that most beginners fail to grasp; any teacher who has tried to define a mass point will know this. The *rigid body* (tinplate) also has an involved definition, but it is one that can be made to *seem* plausible. Besides, the forces acting on a mass point can be shown to be a special case of the forces acting on a rigid body. Why then favor the special case when the generalization is just as easy to understand?

WHAT IS A TINPLATE?

This experiment is one of a group of three which intends to interpret the concept of force. The tinplate simulates a study of three forces that act on a *rigid body;* it is intended to replace the familiar force board experiment (shown in Fig. 1).

The additional cost of a tinplate experiment is extremely small because it utilizes practically all of the force board apparatus. Additional materials include such items as string, two metal pins, center punch, sheet metal cutter, hammer, small drill, and a 20 cm×30 cm sheet of galvanized iron from which the tinplate is made. The tinplate is shown in Fig. 2; it virtually replaces the *mass point.*

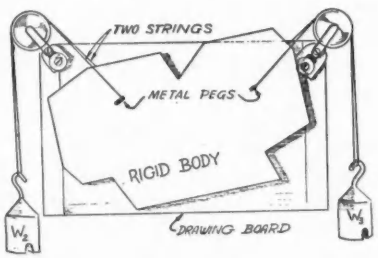

FIG. 2. The tinplate. Each of the supporting strings is doubled and looped over the front and back ends of a metal peg. This keeps the problem in two dimensions.

The tinplate is more or less an arbitrarily shaped object of about 160 g mass when it is cut out of a piece of 26 gauge galvanized iron. The tinplate is prepared for experimentation as follows:

1. Two holes are drilled in the tinplate; their relative positions appear in Fig. 2. Metal pegs are forced into these holes so that strings fastened to these pegs can, with the aid of weights W_2 and W_3, support the tinplate.

2. The tinplate is now balanced on the end of a center punch; a hole is drilled through the balance point. This locates the center of gravity of the *rigid body.* (Not shown in Fig. 2.)

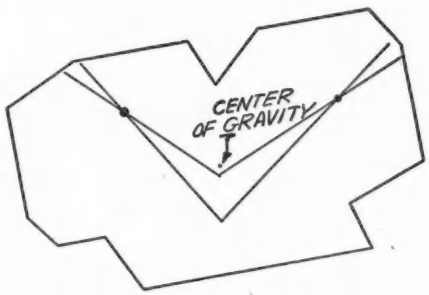

FIG. 3. Typical set of data. Weight of tinplate is 162 grams. $W_2=150$ grams; $W_2'=120$ grams; $W_3=150$ grams; $W_3'=100$ grams.

3. A blank sheet of paper (not shown in Fig. 2) is cut so that it has about the same shape as the tinplate; it is scotch taped to the tinplate. This paper becomes the student's *data sheet.*

4. The tinplate is suspended by strings which pass over pulleys as shown in Fig. 2.

ACQUIRING THE DATA

It is necessary to transcribe the direction of each string upon the data sheet which is fastened to the tinplate. With the tinplate hanging from the pulleys data is obtained as follows. Place two wooden wedges behind and on each side of the tinplate; adjust them until the plate rests lightly against these wedges. Now an assistant can press the plate firmly against the wedges, and at the same time transcribe the directions of the strings on the data sheet. By using a different set of weights W_2' and W_3' another set

of directions made by the strings can be transcribed on the data sheet; when the two sets of directions are extended, the result looks like that shown in Fig. 3.

Before the data can be ready for interpretation, the tinplate must be weighed by means of a balance.

DESCRIPTION OF DATA

It is important for both teacher and student to recognize that it is first necessary to *assume* that the data in Fig. 3 can be described in terms of vectors and later to prove that this assumption is correct. The data, which are assumed to be described completely by vectors, are the forces in the strings which support the tinplate. Figure 4 illustrates how precisely the data of Fig. 3 can be described by graphical means. Note:

$$F_2 = W_2 \qquad F_2' = W_2'$$
$$F_3 = W_3 \qquad F_3' = W_3'$$

are approximations which improve when pulleys having very little friction are used.

Vectors $\mathbf{F}_2$ and $\mathbf{F}_3$ describe two of the three forces that act on the tinplate; the third force is the force due to gravity, which can be described by $\mathbf{F}_1$. The existence of $\mathbf{F}_1$ is not at once apparent to the average student; it actually does tax his powers of imagination. The experiment has now been reduced to the study of three nonparallel, concurrent forces acting on a rigid body at rest and having the *assumed* property of being described by vectors.

INTERPRETATION OF DATA

Once the data have been recorded in the graphical vector form, it is most natural to compare the vector sum $-\mathbf{F}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ with the vector sum $-\mathbf{F}_1'$ of $\mathbf{F}_2'$ and $\mathbf{F}_3'$. The reason for the negative sign is apparent when one anticipates that $-\mathbf{F}_1$ and $-\mathbf{F}_1'$ are *equal and opposite* to $\mathbf{F}_1$ (force due to gravity); this notation conforms with standard practice in vector analysis; that is, $\mathbf{A}$ is equal and opposite to $-\mathbf{A}$. Figure 5 shows the operation of vector addition which gives the resultant $-\mathbf{F}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ and the resultant $-\mathbf{F}_1'$ of $\mathbf{F}_2'$ and $\mathbf{F}_3'$. Symbolically one writes

$$-\mathbf{F}_1 = \mathbf{F}_2 + \mathbf{F}_3 \qquad (1)$$
$$-\mathbf{F}_1' = \mathbf{F}_2' + \mathbf{F}_3'. \qquad (2)$$

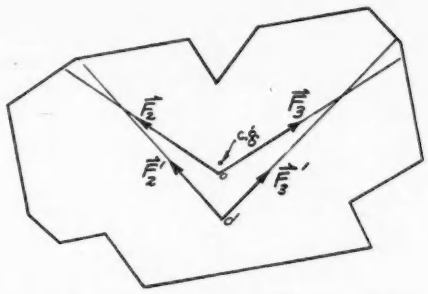

FIG. 4. Scale: 1 cm = 40 grams-force. $F_2 = 150$ grams; $F_2' = 120$ grams; $F_3 = 150$ grams; $F_3' = 100$ grams.

One also observes from Fig. 5 that $-\mathbf{F}_1$ and $-\mathbf{F}_1'$ are approximately equal and the plumb line indicates that the force due to gravity is parallel to $-\mathbf{F}_1$ and $-\mathbf{F}_1'$.

INTERPRETATION OF RESULTS

1. It is very significant to note that the resultants $-\mathbf{F}_1$ and $-\mathbf{F}_1'$ have magnitudes that are practically equal to the weight of the tinplate, i.e.,

$$F_1 = F_1' = 164 \text{ g (162 g = weight of tinplate).}$$

Theoretically, they are exactly equal to the weight of the tinplate.

2. Equations (1) and (2) have a profound physical interpretation; the equality sign implies the following: $-\mathbf{F}_1$ may replace or be a substitute for $\mathbf{F}_2$ and $\mathbf{F}_3$. A student can convince himself of this interpretation by drilling a hole in the tin-

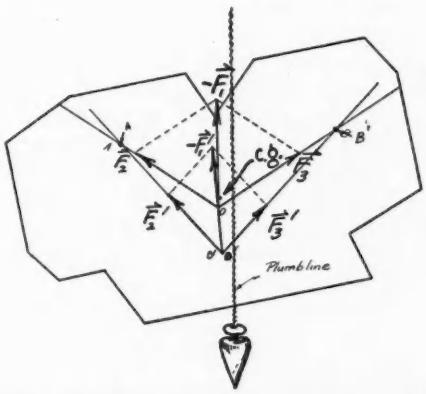

FIG. 5. $F_1 = F_1' = 164$ grams. In practice the center of gravity lies close to line OO'; theoretically, of course, the center of gravity lies on line OO'.

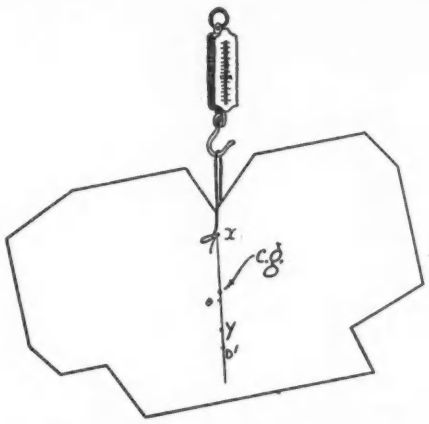

FIG. 6. This diagram shows that $\mathbf{F}_2$ and $\mathbf{F}_3$ have been replaced by $-\mathbf{F}_1$, where $-\mathbf{F}_1$ is the force exerted by spring balance and experimenter's hand.

plate anywhere along the line OO', threading a string through this hole, looping it over the hook of a spring balance, pulling in the direction of $-\mathbf{F}_1$ or $-\mathbf{F}_1'$ on OO' while his laboratory partner removes the tension in the strings which are fastened to the metal pegs at A and B. The equilibrium of the tinplate is not disturbed if the resultant *takes over* at the same time that $\mathbf{F}_2$ and $\mathbf{F}_3$ *quit their jobs*. The student may now read the spring balance; it reads 164 g. He is obviously applying $-\mathbf{F}_1$.

3. It is now apparent that one has proven experimentally that forces can be described by vectors. This proof results from the fact that there have been performed two independent experiments which produce the same conclusion. One of these experiments was performed on the basis of an assumption (that forces are described by vectors and are added like vectors); the conclusion was that $F_1=164$ g *or* the tinplate *weighs* 164 g. This *same* conclusion was obtained by another independent experiment—that of simply weighing the tinplate on a balance. It follows, then, that vector mathematics must be applied to forces.

4. Transposing $-\mathbf{F}_1$ in Eq. (1), one gets:

$$0=\mathbf{F}_1+\mathbf{F}_2+\mathbf{F}_3,$$

which is the first necessary condition imposed on a rigid body in order that it be at rest. It is well to note that $\mathbf{F}_1$ and $-\mathbf{F}_1$ differ in several respects: (1) $\mathbf{F}_1$ is opposite to $-\mathbf{F}_1$, (2) $\mathbf{F}_1$ is caused by the earth; $-\mathbf{F}_1$ is caused by the student, spring balance and the string, only when he is verifying the physical interpretation of Eqs. (1) and (2).

5. In general, a force has three properties: direction, magnitude, and an action line which is not emphasized in most sophomore physics courses. One defines an action line to be the locus of all possible points of application; the locus of such points is a straight line.

6. By drilling holes in the tinplate at points x and y as shown in Fig. 6, the student can be shown, by the proper application of $-\mathbf{F}_1$ or $-\mathbf{F}_1'$, the three types of equilibrium: (a) Threading a string through the hole at x, applying $-\mathbf{F}_1$, and jarring the tinplate into a small arbitrary rotational displacement about x gives evidence of stable equilibrium. The plate returns to the position it had prior to the jarring. (b) Next, by threading the string through the center of gravity, by applying $-\mathbf{F}_1$, and by jarring the tinplate about the center of gravity one is given evidence of a peculiar type of stability which can be interpreted as follows: the tinplate is constrained to rotate about the center of gravity; it is suspended on a string from that point. It is clear that the potential energy is a constant for all orientations of the plate. The potential energy function, therefore, has no minimum so that there is no *unique position* of stable equilibrium. This statement can be made less frightening to the beginner of physics. (c) By making the hole at y the point of application for $-\mathbf{F}_1$, one has evidence of unstable equilibrium.

THE TINPLATE AS A LECTURE-DEMONSTRATION

With a slight modification this experiment has been used effectively in class demonstrations for both the introductory and intermediate physics courses. Instead of suspending the plate by means of a force board, it is hung directly in front of the blackboard by means of pulleys screwed into the upper framework of the blackboard.

Thermometers of the Royal Society, 1663–1768*

LOUISE DIEHL PATTERSON
Wheaton College, Norton, Massachusetts
(Received February 19, 1951)

Between 1663 and 1665, Robert Hooke designed a thermometer for the Royal Society which served as a standard for the graduation of other seventeenth-century instruments. This paper describes the construction and scale of the standard, the use of Hooke's scale in meteorological diaries of the period 1669 to 1709, and its modification by Francis Hauksbee, the Younger, in the early eighteenth century. It includes a suggestion for interpretation of the barometric observations in Hooke's Guildhall diary. It points out the influence of the Royal Society's meteorological observations in the age of Fahrenheit, Réaumur, and Celsius.

PRIOR to the Stuart Restoration of 1660, the thermometer used in England was a qualitative instrument, the old-fashioned weatherglass such as that of Robert Fludd (Fig. 1). In 1661, Sir Robert Southwell brought a small sealed spirit-thermometer from Italy, and the new invention quickly supplanted the open weatherglass.[1,†] Robert Hooke, Curator of the Royal Society, modified the design of the Italian instrument into the long bolthead thermometer (Fig. 2), introduced the use of dye for the fluid, and provided it with a scale. Although Hooke's scale was by no means universally adopted, a number of instrument makers and observers used the Royal Society's thermometer as a standard, and a small body of meteorological data has survived in which the temperatures were recorded in Hooke's units.

This statement is contrary to the general belief that no standardization of the thermometer occurred prior to Fahrenheit's work in the eighteenth century. The start of the Fahrenheit scale has been placed at 1708, the year of the Great Frost, when Fahrenheit visited Ole Rømer at Copenhagen, found him graduating his thermometers from two fixed points, and returned to Amsterdam to transform Rømer's scale with gradual modification into the Fahrenheit scale as we know it today.[2] The following account of the Royal Society's thermometers deals primarily with their history in the half-century between Southwell's introduction of the sealed thermometer into England and the appearance of the Fahrenheit scale in Holland, and more briefly with their influence in the eighteenth century.

EVIDENCE FOR THE EXISTENCE OF A STANDARD

Evidence that the Royal Society's thermometer was used as a standard appears in the writings of Sir Robert Moray, John Locke, Robert Boyle, and Hooke himself. Moray's letters to Christian Huygens[3] describe a thermometer which the operator to the Royal Society built for Huygens in 1665 and graduated by Hooke's standard. John Locke[4] recorded in his journal for December 1669, "I began to keep a Register

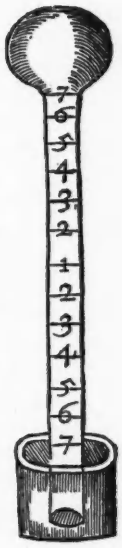

FIG. 1. Weatherglass used in England before the Stuart Restoration. Robert Fludd, *Mosaical Philosophy* (London, 1659).

* Adapted from a paper read at the History of Science Society, Cleveland, December 29, 1950. Material to appear in *Isis* under the title, "The Royal Society's Standard Thermometer, 1663–1709," is included by courtesy of the Editor, Professor George Sarton.

† References will be found at the end of the paper under heading "Bibliographical Notes."

FIG. 2. Sealed spirit-thermometer of the bolthead type used in the Restoration period. Robert Boyle, *New Experiments Touching Cold* (London, 1665).

of the Heat and Cold at London, by a Thermometer of the Royal Society's Standard." Boyle in 1683[5] and 1684[6] several times mentioned a "gag'd" thermometer, or "trusty sealed Thermoscope that was made by the Standard-weather-glass at Gresham Colledge," which he said he had used for some years. Hooke in 1681[7] reported that all the thermometers built by Richard Shortgrave were graduated by a standard such as Hooke had recommended. The weather-clock[8] which Thomas Crawley built for the Royal Society in 1678, and the marine-barometers[9] which Henry Hunt offered for sale toward the end of the century included spirit-thermometers graduated by the standard.

Hooke's comments show that he tried to devise a standardization process which could be duplicated independently of the instrument. In practice, however, the Royal Society's thermometer rather than Hooke's process served for the graduation of new instruments.

OTHER EARLY STANDARDS

The Gresham College thermometer, used by the Royal Society from 1665 to 1709, was not the only standard thermometer in this early period. The Royal Society had a standard of sorts by 1663,[10] and Hooke had used such instruments experimentally for several years before he produced the Gresham thermometer.[11] Various London makers of thermometers kept individual standards a little later in the century. And the Gresham instrument must be distinguished from the Royal Society's eighteenth-century standard,[12] which may perhaps have been the same thermometer, but which had a modified scale. There were thus at least three successive standards of the Royal Society, plus those of Hooke and the London instrument makers, used in England prior to widespread standardization which occurred in the course of the eighteenth century.

EVOLUTION OF THE STANDARD

Construction of such an instrument grew out of the Royal Society's Baconian project to compile a history of the weather. Hooke proposed this scheme at a meeting in February, 1663,[13] but was diverted from its immediate prosecution by the abortive trials of Huygens' pendulum-clocks[14] as marine chronometers aboard the *Katherine-yacht*, the King's pleasure-boat, in early March of that year. On the expedition, however, Hooke experimented with a rudimentary registering thermometer attached to a sounding instrument, which informed him that the water on the floor of the Channel did not reach "the degree of extreme cold."

John Wilkins[15] reminded the Society of its plan for a history of the weather in September of 1663. A month later the standard appeared in a list of "observables" which Hooke recommended:[16]

> The heat and cold of ye air, with its degrees and continuances, which may be observ'd by a good thermometer seald up soe that it may serve for a constant standard for Winter and Summer from year to year, as long as the Observations be made.

In the fall of the same year the operator to the Royal Society built a number of identical thermometers for members who had volunteered to observe the weather.[17] One was to be kept at the Society as a standard, but it was soon followed by the instrument which came to be known as "the standard of Gresham College."

The extent of this undertaking by a generation of men who had grown up without thermometers is revealed by Boyle's suggestion of December 1663:[18]

> Mr. Boyle desired also, that inquiry might be made, whether there be not greater degrees of cold than is neces-

sary to glaciation? And whether water turned into ice may not be advanced to a greater degree of cold.

The problem at this stage was not the selection of appropriate fixed points, but how without fixed points to determine whether the freezing point of water was or was not fixed at absolute zero. Hooke at first tried to use two freezing points, one the temperature at which a water-bath froze at the surface, and the other that at which it was completely congealed.[19]

THE GRESHAM COLLEGE STANDARD OF 1665–1709

Hooke's *Micrographia*, printed in October of 1664, gives a detailed description of the method which he finally adopted for graduation of the standard.[20] At a Royal Society meeting of 11 January 1665, it was recorded:[21]

> Mr. Hooke produced his thermometrical standard for heat and cold, and gave an account how it had been made, *viz.* after the manner described in his *Micrographia*. This was looked upon, though not exact, yet better than the other ways hereto used.

Such was the doubtful welcome of the standard of Gresham College.

The process described in the *Micrographia* was essentially that employed by René de Réaumur in the following century. Hooke used a cylindrical standardizing vessel with a smaller cylinder projecting out of the top (Fig. 3). The small cylinder had a diameter equal to one-tenth that of the large, and was marked at intervals equal to one-tenth the depth of the larger vessel. He used the same fluid in the thermometer and in the standardizing vessel, and heated them together in a bath of distilled water in order to locate heights of the thermometer column showing equal increments in the volume of the spirit. As he described the process in the *Micrographia* a division of the smaller cylinder represented an expansion of one-thousandth above the volume of the fluid at freezing. In practice, Hooke must have numbered every other division, and his degree was roughly equal to two modern Réaumur degrees.

Hooke placed the origin of his scale at the temperature at which a chilled water-bath began to freeze and shoot into flakes as he stirred the surface with the point of a needle. Circumstantial evidence, to be discussed later, indicates

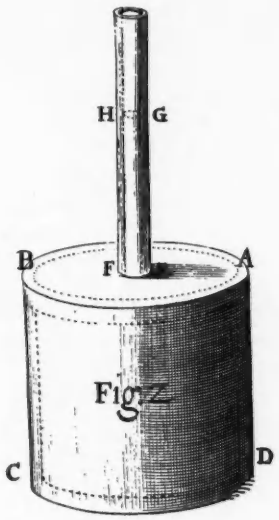

FIG. 3. Vessel used for graduation of the Royal Society's standard thermometer, Robert Hooke, *Micrographia* (London, 1665).

that his failure to notice the anomalous expansion of water near the freezing-point probably introduced a zero error of about two Fahrenheit degrees. William Croune gave experimental demonstrations of this anomaly at the Royal Society in February, 1684.[22]

HOOKE'S THERMOMETRIC UNIT

The approximate size of Hooke's degree can be recovered from the scale of his marine barometer, which Edmond Halley used on the Atlantic voyage of the *Paramore Pink* and described in the *Philosophical Transactions*.[23] The instrument consisted of an old-fashioned weather-glass or inverted open bolthead, acting as an air-thermometer, together with a sealed spirit-thermometer. The error of the air-thermometer provided a measure of the barometric pressure, which was read directly from a sliding scale. (Fig. 4). Amonton later described such an instrument at the Royal Academy;[24] in 1816, A. Adie, a Scottish optician, patented Hooke's invention as the "Sympiesometer," under which name it was used for navigation in the nineteenth century.[25]

Halley's diagram shows that, starting at Hooke's standard conditions of freezing temperature and 29.5 inches of mercury, an increase of 11.6° in temperature and of 3 inches in barometric pressure would leave the volume of the

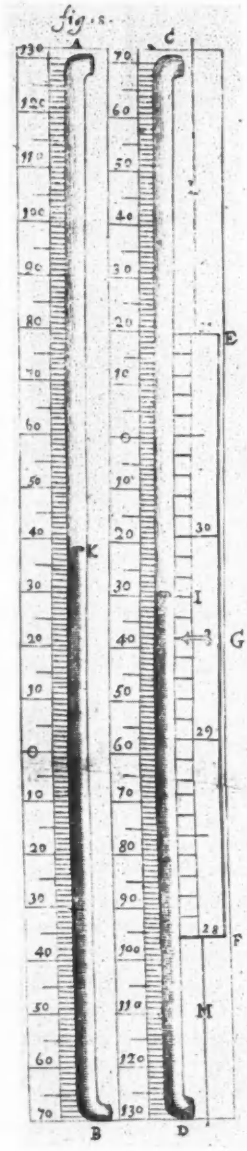

FIG. 4. The Royal Society's thermometric scale applied to the marine-barometer, 1667, and used on the South Atlantic voyage of the *Paramore Pink*, 1699–1701. Scales: *AB*, spirit-thermometer; *CD*, air-thermometer; *EF*, sliding barometric plate of the air-thermometer. *Philosophical Transactions*, February, 1701.

confined gas unchanged. Since the density of the confined air was a fraction of that of the atmospheric sample represented by the barometric slider, Hooke's thermometric unit must have been about 2.4C° or 4.3F°. Halley gave a numerical example, showing the volume of air at 3.8° and 29.7 inches of barometric pressure equal to that at 3.0° and 29.5 inches, which gives the same result. From Halley's discussion, it does not appear that the volume of the gas was kept constant—Amonton permitted the volume in his marine-barometer to vary because he found that the resulting error was small, and for marine use preferred simplicity of operation to accuracy—but the scale of Hooke's instrument no doubt applied to a volume at or near his standard conditions.

Hooke invented the marine-barometer in 1667 and it evolved through a long period of experimentation, during which the individual instruments were graduated empirically. At the outset he had suggested preparing a table to be used with each instrument; the invention of the slider no doubt grew out of such tables. Various observers compared the performance of the marine barometer with that of the ordinary mercury barometer, and found it satisfactory except in the case of old instruments, where the readings were somewhat high. Halley's scale therefore gives the best available information as to the size of Hooke's unit, and values of 2.6C° and 2.1C° obtained from two patently inaccurate expansion experiments of Francis Hauksbee, the Elder, may be discounted in favor of the 2.4C° from Hooke's instrument.

Boyle reported that in summer, when the weather was very hot, "the tincted spirit" of his gauged thermoscope ascended to the eighth, ninth, and sometimes almost to the tenth mark. Fahrenheit equivalents of 67°F, 71°F, and 75°F are not unreasonable in view of the present mean July temperature of London, 64.4°F.

THE SCALE OF THE STANDARD THERMOMETER

In the following discussion, Hooke's name or initial applies to the Gresham College or seventeenth-century scale, and the initials RS of the Royal Society to that of the eighteenth century. Except as otherwise indicated, Fahrenheit equivalents are not corrected for a zero error of probably 2F° at the freezing point of the early thermometers of Fahrenheit and of the Royal Society.

The distinguishing characteristic of Hooke's thermometer is its range, from −7°H at "extreme cold" to 13°H for the greatest summer heat of the English climate. The 13°H at the upper limit was a temperature which Hooke recorded

for 16 June 1676, "the hottest day of all" in what was known as "the extreme hot time in London."[26] The equivalent indicated by the marine barometer is 88°F.

Various eighteenth-century writers placed the origin 0°RS of the eighteenth-century standard at 88°F—sometimes 89°F. According to William Derham, Cromwell Mortimer, George Martine, and others, its freezing point was at 65°RS; the scale extended downward to 100°RS.[27] Francis Hauksbee, the Younger, author of the eighteenth century scale, must therefore have adopted the 13°H at the upper limit of the old scale as his origin, and used one-fifth of Hooke's unit as the degree RS. A diagram (Fig. 5) of one of the early Hauksbee thermometers, showing a scale of 23 divisions, suggests that Hauksbee at first used Hooke's original unit.[28]

DESIGN AND USE OF THE THERMOMETERS

A thermometer of Hooke's design[29] was from 2 ft to 4 ft long and of 0.1 inch bore or less. The "Body" or lower bulb was 2 inches in diameter, and the "Head" or upper bulb somewhat less. Hooke partly filled the cavity with alcohol colored with Brazil-wood or cochineal and then sealed on the head with a lamp. To secure a tight seal, he had the bulb especially made at the glass-house of the same material as the stem, which he selected carefully for straightness and even bore.

The head served as a safeguard against bursting in case of unpredictable hot weather or even less predictable experimentation. Boyle's gauged thermoscope successfully withstood the temperature of a young gentlewoman, who was bled into a porringer containing the instrument by a surgeon of Boyle's acquaintance, but overflowed into the upper ball, "considerably above the usual Warmth of the Air in the Dog-days," in a similar experiment with a middle-aged man.[30]

Meteorological thermometers of the Royal Society habitually were kept indoors near a window in an unheated room, in order that they might encounter representative weather and be protected against accident. William Derham about 1699 initiated the practice of taking the thermometer outdoors, but even toward the middle of the eighteenth century the Society's observers ordinarily measured the temperature in unheated rooms.[31] Comments of Boyle and Moray show that such indoor measurements differed in freezing weather from outdoor temperatures by a little over 2H° or about 9F°.[32]

METEOROLOGICAL OBSERVATIONS

Journal of John Locke, 26 December 1669 to 14 June 1675.[33]—Several meteorological diaries have been found to give temperatures in Hooke's units, of which the earliest is the intermittent record kept by John Locke at London for the period December, 1669, to June, 1675. Locke's temperatures, probably observed at Exeter House in the Strand where he lived, range from equivalents of 19°F (−3°H) in December 1669 to 72°F (9.2°H) in June 1672. Locke estimated to a quarter of Hooke's degree—approximately the nearest Fahrenheit degree—and from May 1670 onward used Hooke's quarter-degree as his unit.

Locke's tract on the *Elements of Natural*

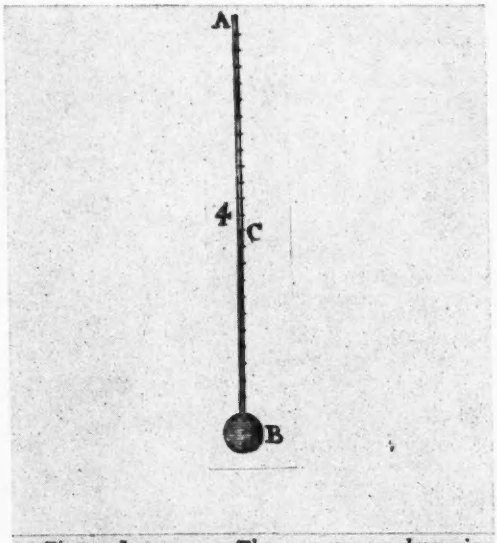

Fig. 4. Is a common Thermometer, to determine the Quantity of the Heat of the Air, or of any Liquor, by the Rarefaction of Spirit of Wine contain'd in the hollow Ball at the Bottom, and its Conſequent aſcending to the ſeveral Diviſions on the ſmall Tube.

FIG. 5. Early Hauksbee thermometer, probably graduated by the Royal Society's standard in the early eighteenth century. Francis Hauksbee (the Younger) and William Whiston, *Course of Mechanical, Optical, Hydrostatical, and Pneumatical Experiments* (London, 1714).

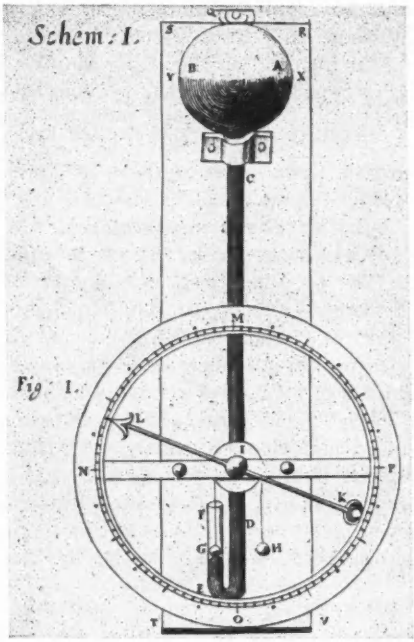

FIG. 6. Wheel-barometer used in Hooke's meteorological observations. Robert Hooke, *Micrographia* (London, 1665).

Philosophy[34] included an improvement over Boyle's suggestion concerning absolute zero. He proposed that "the utmost degree of cold is the cessation of that motion of the insensible particles, which to our touch is heat."

Journal of Robert Hooke, 10 March 1672 to 27 April 1673.[35]—The next journal is that kept by Robert Hooke at Gresham College, March, 1672, to April, 1673, with scattered later entries. This diary has been regarded as unintelligible, but is undoubtedly based on the Gresham standard. Hooke's temperatures range from freezing to the 13°H of "the hottest day of all," and like those of Locke were recorded to the nearest quarter of his degree.

His measurements overlap with those of Locke on eleven days. The temperatures noted by Hooke at an unspecified hour in the eastern part of London differ at most from those observed by Locke in the west by about 7F°. The average difference, as given by the marine-barometer factor, is 3F°. This occurred less than ten years after Boyle had questioned whether absolute zero were at freezing, and some years before the birth of Fahrenheit, Réaumur, and Celsius.

For interpretation of Hooke's barometric pressures, it is necessary to know the scale of his wheel-barometer (Fig. 6). His unit was 0.01 inch of mercury, and a complete turn of the wheel represented 2.0 inches.[36] Hooke used ascending numerals for a falling column; the origin of his circular scale apparently coincided with a reading of 200 at his standard pressure of 29.5 inches. A quarter-turn of the wheel from 200 to 150 thus showed a rise in pressure from 29.5 inches to 30.0 inches, and a quarter-turn from (2)00 to 50 showed a drop from 29.5 inches to 29.0 inches.

The following are characteristic diary entries from the winter and spring of 1673:[37]

February 7th—a very cold black frost. the sun broke through the fogg. Wind E. ☿ 125. Th. 0 3/4.

April 16th—an exceeding lovely May day. Wind S.E. ☿ 180. Th. 7. being the first summer morning and finding my blood begin to ferment I walkd till noon in the feild with Blackburne.

Journal of Henry Hunt, 10 November 1692 to 9 November 1693.[38]—Edmond Halley reported the observations of Henry Hunt, operator to the Royal Society, in the *Philosophical Transactions* for 1694. Hunt's journal is unbroken for the year, 1692–1693, and his measurements were all made at Gresham College at eight o'clock in the morning. The temperatures, given in Hooke's tenth-degrees, range from an equivalent of 25°F (−1.7°H) in November to 70°F (8.7°H) in June. Hunt some years earlier had been placed in charge of the Society's weather-clock, and his journal may have been compiled from some of the papers upon which it made automatic recordings.

Journal of Edmond Halley, 27 September 1699 to 10 March 1700.[39]—The most colorful of the journals referred to Hooke's scale was that kept by Edmond Halley on the voyage of the *Paramore Pink*, September, 1699, to March, 1700. Halley recorded temperatures from the Strait of Dover to Rio Janeiro and south latitude 52° and back to St. Helena. The season proved extraordinarily cold, and Halley found himself at the southern extremity of his voyage, "alone

without a Consort" among mountains of ice, "very hard and white as alabaster," from which he narrowly escaped shipwreck. The temperatures, measured in his cabin in Hooke's tenth-degrees, range from the equivalent of 76°F (10.1°H) near the equator on the outbound voyage in October, to 30°F (−0.4°H) near the southern limit of the voyage in January.

Halley reported that the marine barometer never failed to give advance warning of bad weather. He recorded pressures from 28.7 inches to 30.1 inches, and enumerated in the margin of his diary what appear to have been successive proofs of the marine barometer's performance before storms. Thus the old-fashioned weather-glass, at the middle of the century scarcely more than the perquisite of a mystical philosopher, by the end of the seventeenth century had become a quantitative instrument, tested in the ice-floes of the South Atlantic.

Ole Rømer's journal of the Great Frost, 26 December 1708 to 6 April 1709.[40]—The last journal is that kept by Ole Rømer at Copenhagen during the Great Frost of 1708–1709. Rømer's record, which is in the form of a graph, was reported to the Royal Society, where William Derham commented on it in the *Philosophical Transactions* for 1709. Although Rømer did not state explicitly that he used the Royal Society's standard, there is reason to believe that he used Hooke's unit on a scale with a different origin.

Fahrenheit stated[41] that he saw Rømer in 1708 use as fixed points the temperatures of a mixture of ice and water at 7½°, and of "blood-heat" at 22½° of Rømer's scale. Rømer's degree, as Fahrenheit described it, was identical with that of Hooke.

Rømer's graph shows what appears to be Hooke's scale placed just above his own. (Fig. 7). The units are identical, but the origin of the upper scale is at 8°Ro, slightly higher than Rømer's freezing point, just as it should be if Croune's discovery of the anomalous expansion had led to an improved definition of the freezing point which Rømer used. Since Rømer was a member of the Royal Society and had visited its repository of instruments when he went to England to examine the English standards, it is unlikely that the correspondence between the two scales was accidental.

Rømer is sometimes thought to have used a smaller unit because in 1702 he had proposed the boiling point of water at 60° of his scale as a fixed point for graduation of a spirit-thermometer.[42] As Dr. Dorsey of the Bureau of Standards has pointed out, this early suggestion was impractical; furthermore, Rømer's expansion experiments, as Dr. Dorsey analyzed them, give conversion factors from 1.8C° to over 3C°, a range which would allow for either upper fixed point, or even for the use of more than one scale. It was entirely possible for Rømer to measure "blood-heat" with a spirit-thermometer, however; Fahrenheit placed such an instrument in the mouth or armpit of a healthy man, and the blood of the gentry had been shed on the bulb of Boyle's gauged thermoscope in the preceeding century. Derham's ability to interpret Rømer's graph, which surprised Kirstine Meyer, editor of his *Adversia*, is understandable if Rømer reported his measurements to the Royal Society in the Society's own units.

Rømer's temperatures for the Great Frost all lie between the origin of his scale at about Fahrenheit zero and 8°Ro or 34°F. Derham[43] was skeptical of this record of prolonged freezing until it had been confirmed by other observers, who said that the harbor of Copenhagen froze to a depth of 27 inches and people walked to Sweden on the ice even in April:

William Derham's comparison of Great Frosts, 1709.[44]—While Rømer measured the frost at Copenhagen, William Derham used the Royal Society's thermometer to compare the temperature at London with those in previous great frosts. Derham used Hooke's tenth-degree, and reported the following readings:

(−)4.0°H or 15°F for January fifth in the Long Frost of 1683–84;
(−)4.1°H or 14°F for January twenty-sixth in 1696–97;
(−)4.3°H or 13°F for January third in the Great Frost of 1708–9.

These, according to Derham, were the "greatest descents" of the Royal Society's thermometer. They were all measured by the "self-same" instrument. Hauksbee, the Elder, a little later reported a lower temperature, −5.0°H or 10°F, for an unspecified date.[45]

Although one cannot be sure that the Royal Society used only one instrument as standard in

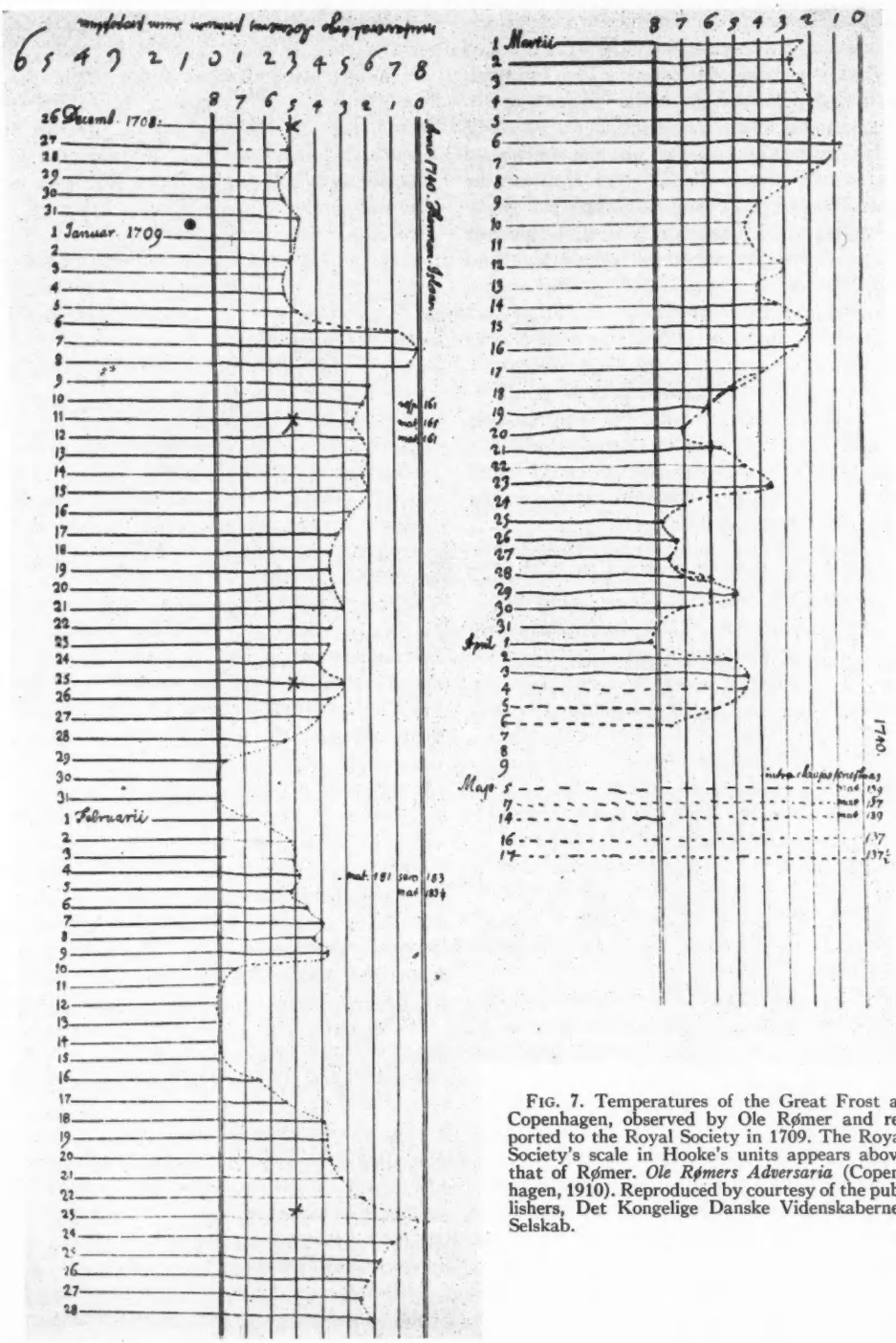

FIG. 7. Temperatures of the Great Frost at Copenhagen, observed by Ole Rømer and reported to the Royal Society in 1709. The Royal Society's scale in Hooke's units appears above that of Rømer. *Ole Rømers Adversaria* (Copenhagen, 1910). Reproduced by courtesy of the publishers, Det Kongelige Danske Videnskabernes Selskab.

the entire period, 1665 to 1709, Derham's report, together with Boyle's discussion, provides some assurance as to the constancy of the thermometric unit. The thermometer used by Derham in the Great Frost of 1709 had been used earlier in the Long Frost of 1683–1684, and Boyle, writing in the year of the Long Frost, gave a description of the performance of his thermometer indoors and outdoors at freezing which coincided with the information about Hooke's standard which Moray had sent to Huygens early in 1665. From the fact that the eighteenth-century standard was called "the ancient thermometer of the Royal Society," it is possible that the later instrument was actually the standard of Gresham College with new numerals on the old scale.

THE INFLUENCE OF THE ROYAL SOCIETY'S THERMOMETERS

The Royal Society carried on the meteorological research in the eighteenth century, with William Derham and James Jurin as its chief promotors, and with such distinguished members as Daniel Gabriel Fahrenheit, Joseph Nicholas de l'Isle, and Anders Celsius among the contributers.[46] Francis Hauksbee, nephew of the elder Hauksbee, built the Royal Society's thermometers and graduated them by the standard kept in "the Society-house" at Crane Court. These instruments were used by the Berlin Academy and the Swedish Royal Society at Upsala, and were widely distributed among observers as far afield as Sicily, South Carolina, Finland, Russia, and Bengal. Hauksbee's thermometers, enthusiastically recommended by Jurin in 1723, eventually were found to be grossly inaccurate; by the middle of the century both English and European observers preferred the expertly fashioned instruments of John Bird of London to those of Hauksbee. As late as 1768, the Scottish instrument maker, James Short, however, was still using indoors an old Hauksbee spirit-thermometer, which he had purchased along with a Hauksbee mercury thermometer in 1738, though he had adopted the Fahrenheit thermometer for use outdoors.[47]

In spite of a deterioration in the quality of the Royal Society's thermometers in the eighteenth century, the Society's history of the weather and widespread popularization of the use of a standard were influential factors in the development of the modern thermometer. A number of early thermometers not referred explicitly to the Royal Society's standard appear to have used imitations and modifications of its scale, with units at least approximating subdivisions of Hooke's degree.

(a) Derham's Thermometer at Upminster

Several thermometers had a twenty-three or twenty-four degree scale, a unit resembling that of Hooke, and the freezing point at 7° or 8°. William Derham used such an instrument for meteorological observations ranging from 6.2° to 17.8° outdoors at Upminster in 1699.[48] Derham placed the freezing point at 8° or 8.2°, and said that the column rose to 22.5° in direct exposure to "a pretty strong sun." Whether or not Derham's thermometer was graduated by the standard, his scale appears to have been the prototype of that of Rømer. (Derham's use of inches as a unit does not preclude the use of a standard, since instrument makers chose dimensions of the bulb and stem in a simple relation to each other and to those of the standard.) What was known as "the Edinburgh thermometer" and used from 1731 to 1737 by the Edinburgh Philosophical Society, which participated in the Royal Society's meteorological observations, had a scale similar to those used by Derham and Rømer, with freezing at 8.2 inches and body temperature at 22.2 inches.[49] Fahrenheit until 1717 used Rømer's scale with both the whole and the quarter-unit.[50] Fahrenheit's tutor, Barnsdorf of Rostock, in 1712 at Berlin, and a little later Barnsdorf's disciple, Johann Lange at Halle, used the same scale, which Lange in turn taught to Lerch and Maas.[51] Lange used a Hauksbee thermometer and contributed observations to the Royal Society in 1728. Chrètien Kirch adopted the Derham-Rømer scale for observations at Berlin in 1715, and Kirch's thermometers were used for simultaneous observations at Berlin and in Pennsylvania in 1731–1732.[52]

(b) The Hanow and Fahrenheit Thermometers

In 1714, Christian Wolff, like Locke and Fahrenheit, divided the unit into four parts.[53] Michael Christian Hanow used the quarter-unit

for observations at Dantzig from 1739 to 1752.[54] Except for Locke, who used Hooke's origin, these men placed 30° at the freezing point and 90° at normal body temperature. Their scale was known as "the Hanow thermometer."

After 1717, Fahrenheit slightly shortened the quarter-unit in order to avoid the use of fractions and conform more nearly to La Hire's thermometer in the Paris Observatory.[55] This modification placed the freezing point at 32° and body temperature at 96°, Fahrenheit's fixed points for the modern scale. Early Fahrenheit thermometers have been found to have "32°" marked at the present 34°F, so that "blood-heat" as Fahrenheit measured it probably was at the present 98°F; early Fahrenheit equivalents of other scales may be similarly in error. Fahrenheit explained the origin of his scale as a temperature observed in Iceland.

(c) Half-unit Scales

John Patrick of London, "the Toricellian operator," used a variety of thermometric scales, one of them said to have had a unit equivalent to that which Hooke had described in the *Micrographia*, but with the origin at "blood-heat" and 32° at freezing.[56] La Court in Holland used the same half-unit in 1716, on an erect scale with the origin selected by Derham, Rømer, and Fahrenheit, and freezing at 15°.[57] De Luc's revision of Réaumur's scale in the middle eighteenth century likewise approximately halved the unit of the Gresham standard and returned it to that described in Hooke's *Micrographia*.[58] On examining Réaumur's thermometers, De Luc found, that as Cromwell Mortimer had suggested, Réaumur had mistaken the boiling point of alcohol for that of water, and inadvertently used a centigrade unit. De Luc enlarged Réaumur's degree, so that the boiling point should fall at 80°R, as Réaumur had intended.

(d) The "Ancient Fahrenheit" Thermometer

One of Fahrenheit's modifications of Rømer's scale, that known as "the ancient Fahrenheit thermometer," used approximately the eighth of Hooke's degree, with the origin at the present 48°F or "temperate," and 90° at the 96°F of the modern scale.[59] Fahrenheit is known to have conferred with Hauksbee on the subject of standards,[60] and there is little doubt as to English influence in his work, through Hauksbee, as well as through Rømer.

(e) The Centesimal Scales

Hauksbee's modification of the Royal Society's scale about 1720 converted Hooke's vigesimal scale with the origin at freezing to an inverted centesimal scale with a unit a little less than 0.5C°, equivalent to the fifth of Hooke's degree. The origin (corrected for an error of 2F° at the freezing point) was at 90°F, "temperate" or 45°RS at 50°F, 65°RS at 34°F, and 100°RS was the lower limit just above the bulb of the standard. Thomas Heath of London used the same scale, extending it downward, however, only to 90°RS.[61]

From the fact that the Royal Society's thermometers were widely used in Sweden from 1725 to 1747, it seems probable that they influenced the invention of the inverted centigrade or Celsius scale, which originated there in 1742. The seventeenth-century Royal Society had recognized the constancy of the freezing and boiling points.[62] The modification of Hauksbee's scale to the inverted centigrade scale required little more than the doubling of the Royal Society's unit and upward displacement of the origin, so that 100° should lie between the two familiar fiducial points. Anders Celsius, who is credited with this invention, in 1739–1740 compared Hauksbee's scale with that of Réaumur and also of de l'Isle,[63] who used an inverted scale with the origin at the boiling point and a 150 degree interval between the boiling-point and freezing. Réaumur's inadvertent use of the centigrade unit may have influenced Celsius' modification, and undoubtedly accounts for some of the obscurity surrounding the invention of the centigrade scale. In any case, after the work of Hauksbee and Réaumur, the appearance of the centigrade scale was almost inevitable, once mercury thermometers were sufficiently perfected to provide a reliable measure of the boiling temperature of water.

There is more definite evidence of the English influence in the invention of the erect centigrade scale attributed to Christin of Lyon, 1743, and to Stroemer of Upsala, 1746. The astronomer

Ferner, in 1761, presented one of John Bird's Swedish thermometers to Mairan at Paris, who described it as resembling that of "Réaumur." Van Swinden believed that Bird used centigrade units on the mysterious "Bird thermometers," which superseded those of Hauksbee in Sweden,[64] and that the "Bird thermometer," as a separate entity, was a thermometric ghost. The Genevan, De Luc, in 1772, listed the centigrade mercury thermometer as "Thermomètre de Londres & de Lyon." He added the centigrade scale to those listed by George Martine as "un autre, qu'on emploie aujourd'hui assez communement à Londres; & qui, depuis quelque tems, est aussi appellé Thermometre de Lyon, parce que M. Cristin l'y a mis en usage."[65] (The centigrade scale was known also in the eighteenth century as "the thermometer of Sweden and Lyon," sometimes as Réaumur's thermometer, whereas the modern Réaumur scale was known as that of De Luc.)

THE HARMONIC ENGLISH THERMOMETER

Thus, after innumerable and sometimes obscure mutations, the origin of the thermometric scale returned to the temperature at which Hooke and his colleagues had placed it in the belief that the freezing-point of water might be absolute zero. Van Swinden suggested that the scale used in Germany by Johann Friederich Weidler, known on the Continent as "the Harmonic English Thermometer," was really the old Hauksbee thermometer—that used by Francis Hauksbee, the Elder, Curator of the Royal Society after Hooke's death; Weidler is known to have used a "Hauksbee" thermometer.[66] If Van Swinden's suggestion is correct, "the Harmonic Thermometer" is the forgotten patriarch among thermometers, for the elder Hauksbee used Hooke's scale, that of the Gresham standard which was father to "the Ancient Thermometer of the Royal Society of London" in the eighteenth century. The "*Scala Gradorum Caloris*" attributed to Newton, in which different fixed points were treated as multiples of the external temperature of the human body and determined by Newton's law of cooling,[67] gave 14 3/11 as "almost the heat of recently shed blood," a fact which suggests that Newton's scale was an adaptation of that of the Gresham standard. Like Rømer, Newton could have determined "blood-heat" more accurately than the higher temperatures on his scale.

CONCLUSION

The significance of the Royal Society's standard thermometer is shown by the size of its degree: $0.2H° = 1RS° = 0.9 - F° = 0.5 - C° = 0.4 - R°$. The Royal Society's unit is a link among the curious numerals, 9, 5, 4, in the conversion factors of the modern scales. The old centesimal standard covered approximately the first 90° of the present Fahrenheit scale, and had a span of 50°C or 40°R. The fact that Rømer, Fahrenheit, and Celsius all participated in the Royal Society's researches, and that Réaumur used the method of graduation described in Hooke's *Micrographia*, a work well known in France, makes it impossible to discount the influence of the Royal Society's standard in the evolution of the modern scales. Although the manner in which that influence operated is not clear in detail, what the inventors of the modern scales did in effect was to fit between convenient fixed points an existing scale which was well known in their day, but which has since dropped from view. The change did not come suddenly, but during the course of a half-century, Fahrenheit, Réaumur, Celsius, and De Luc adopted fiducial points requiring slight change in the fractional part of a unit already in use, derived in Restoration England from the expansion of "the best rectified Spirit of Wine, highly ting'd with the lovely colour of the Cocheneel," and well publicized by the Royal Society's promotion of Hauksbee's instruments; they thus freed themselves from the necessity of referring all instruments to a single standard.

Although it has been thought that there were no comparable measurements of temperature prior to the eighteenth century, measurements made in different parts of London simultaneously and on the same scale are of record in Locke's and Hooke's diaries for 1672. From the rather meager data available, it appears that in the seventeenth century the Royal Society's standardized thermometers were few but skillfully made and carefully used, though indoors, in order to obtain comparable measurements. With mass production in the eighteenth century the

quality of the English instruments declined, and leadership passed to the Continent, where Fahrenheit started building better instruments, and introduced the use of mercury as the thermometric fluid. By the middle of the century, Bird again in London was building "excellent" thermometers, using the Fahrenheit and also the centigrade or "false Réaumur" scales.

Nearly a century elapsed between the invention of unstandardized spirit-thermometers in Italy, and general adoption of the modern scales with two fixed points. At the start of this period the most able scientists had less quantitative knowledge of temperature than a child has today. A short time after the founding of the Royal Society, however, Hooke designed a thermometer with a scale which could be duplicated, and the Society started assembling meteorological observations from all over the world. The Royal Society retained essentially the same scale till the middle of the eighteenth century and collected a voluminous "history of the weather." Much of the great mass of surviving data referred to this early standard is clearly inaccurate. The "Ancient Standard" deserves to be remembered, however, as the backbone of whatever consistency may be found among the vagaries of the early meteorological records, and as a progenitor of the modern thermometer.

BIBLIOGRAPHICAL NOTES

THERMOMETERS OF THE ROYAL SOCIETY, 1663–1768

[1] Robert Hooke, *Posthumous Works* (London, 1705), p. 556. R. T. Gunther, *Early Science in Oxford* (Oxford, 1920–45), Vol. 7, p. 729; Trans. Roy. Soc. (London) **22**, no. 269, 788 (1701).

[2] Kirstine Meyer, Nature **82**, 296–298 (1910); Nature **139**, 585–586 (1937). Carl Boyer, Sci. Monthly **57**, 442–452 (1943). I. Bernard Cohen, Isis **31**, 327–379 (1940), reprinted by the Burndy Library, 1944; Isis **39**, 56–58 (1948). N. Ernest Dorsey, J. Washington Acad. Sci. **36**, 361–372 (1946). J. Newton Friend, Nature **139**, 395–398, 586 (1937).

[3] Christian Huygens, *Oeuvres Complètes* (The Hague, 1888–1950), Vol. 4, p. 429; Vol. 5, pp. 138, 158, 168–169, 188, 228, 245, 344, 345, 360, 505, 551; Vol. 6, p. 11.

[4] Robert Boyle, *General History of the Air* (London, 1692), pp. 116, 118.

[5] Robert Boyle, *New Experiments Touching Cold* (London, 1683), appendix, p. 17, (XVIIIth Title).

[6] Robert Boyle, *Memoirs of Human Blood* (London, 1683–4), pp. 27–30, 41–43.

[7] See Gunther, reference 1, Vol. 7, p. 570.

[8] See Gunther, reference 1, Vol. 6, p. 232; Vol. 7, pp. 505, 518–520. Thomas Birch, *History of the Royal Society* (London, 1756), Vol. 1, pp. 341, 342; Vol. 2, p. 1. R. S. Hooke MS 80, fol. 2, microfilm examined by courtesy of the Royal Society.

[9] Trans. Roy. Soc. (London) **22**, no. 269, 794 (1701); **31**, no. 366, (1720). Hooke, reference 1, p. 556. Robert Boyle, *Works* (London, 1744), Vol. 5, pp. 378, 379.

[10] Birch, see reference 8, Vol. 1, pp. 320, 334, 337.

[11] Huygens, see reference 3, Vol. 5, p. 169.

[12] George Martine, *Essays on Thermometers* (Edinburgh, 1792), p. 46. Louis Cotte, *Traité de Météorologie* (Paris, 1774), pp. 130–131. Tiberius Cavallo, *Elements of Natural Philosophy* (London, 1803), Vol. 3, p. 20. Jan H. Van Swinden, *Dissertation sur la Comparaison des Thermomètres* (Amsterdam, 1778), p. 226.

[13] See Gunther, reference 1, Vol. 6, pp. 114–115. For Christopher Merret's preliminary experiments in 1662, see Boyle, reference 5, "An Account of Freezing," pp. 17, 20.

[14] See Gunther, reference 1, Vol. 6, p. 120; Vol. 8, p. 148. Robert Hooke, *Philosophical Experiments* (London, 1726), pp. 4–6, 105, 226. Abraham Hill, *Familiar Letters* (London, 1767), pp. 91–108.

[15] Gunther, reference 1, Vol. 6, p. 152.

[16] R. S. Hooke MS 23, microfilm examined by courtesy of the Royal Society.

[17] Birch, see reference 8, Vol. 1, pp. 311, 320.

[18] Birch, see reference 8, Vol. 1, p. 337.

[19] Gunther, see reference 1, Vol. 6, p. 24. Huygens, reference 3, Vol. 5, pp. 169, 188, 228, 345.

[20] Gunther, see reference 1, Vol. 6, p. 223; Vol. 13, pp. 38–39. Hooke, see reference 1, p. 556.

[21] Gunther, see reference 1, Vol. 6, pp. 233, 234.

[22] Gunther, see reference 1, Vol. 7, pp. 642–643, 650–651.

[23] Trans. Roy. Soc. (London) **22**, no. 269, 791–794 (1701); **31**, no. 366, (1720). Boyle, reference 9, Vol. 5, pp. 378, 379. Gunther, reference 1, Vol. 6, pp. 323, 324; cf. also Vol. 13, p. 236.

[24] J. A. De Luc, *Recherches sur l'Atmosphère* (Geneva, 1772), Vol. 1, pp. 32–35. Amonton recognized the priority of Hooke's researches on the air-thermometer and mentioned Hooke's influence on Hubin, who built Amonton's instruments. Amonton's inspiration came directly from Huygens' proposal in 1672 of a two-fluid barometer, a suggestion of Descartes which Hooke already had carried out in 1668. See in this connection: Guillaume Amonton, *Remarques et Experiences Phisique* (Paris, 1695), pp. 145 ff., esp. p. 147; Gunther, reference 1, Vol. 6, p. 339, and Vol. 7, pp. 682–688, 692; Huygens, reference 3, Vol. 7, p. 288, and *passim;* Trans. Roy. Soc. (London) **16**, no. 185, 241–244 (1686).

[25] Edward Riddle, *Dr. Hutton's Philosophical Recreations* (London, 1854), p. 658. George Leoni, *Ample Instructions for the Barometer and Thermometer* (New York, 1841), pp. 14, 17–18.

[26] Robert Hooke, *Diary 1672–1680* (London, 1935), p. 237. See Birch, reference 8, Vol. 4, p. 81.

[27] *Philosophical Transactions···Abridged*, John Lowthorpe, Benjamin Motte, John Eames, and John Martyn, ed. (London, 1716–1756), Vol. 8, Pt. 2, p. 574; Vol. 10, Pt. 2, p. 438. Martine, reference 12, p. 46.

[28] Francis Hauksbee and William Whiston, *Course of Mechanical Lectures* [(London, date 1714 or later; see Whiston's *Memoirs*, 1749), Vol. 1, p. 235.]

[29] See Gunther, reference 1, Vol. 13, p. 38; for the properties of the fluid, cf. Vol. 13, pp. 219–220. Huygens, reference 3, Vol. 5, p. 168.

[30] See Boyle, reference 6, pp. 27–29.

[31] *Phil. Trans. . . . Abr.*, reference 27, Vol. 5, p. 98, and *passim.*

[32] Huygens, reference 3, Vol. 5, p. 345. Boyle, reference 5, appendix, pp. 17–18.

[33] Boyle, reference 4, pp. 116–121.

[34] John Locke, *Philosophical Works* (London, 1877), Vol. 2, p. 494.

[35] Hooke, *Diary 1672–1680*, pp. 3, 18–41, and *passim*, and microfilm of the manuscript belonging to the Corporation of London, examined by courtesy of the Guildhall Library.

[36] Gunther, see reference 1, Vol. 13, "The Preface."

[37] Hooke, see reference 26, pp. 27, 39.

[38] Edmond Halley, "Account of the evaporation of water," Trans. Roy. Soc. (London) **18**, no. 212, pp. 183–190 (1694). I am indebted to Mr. D. C. Martin of the Royal Society's Library for the information that several efforts to trace the automatic recordings of the weather-clock, preserved by the early Royal Society, have as yet proved unsuccessful. Cf. Gunther, reference 1, Vol. 7, pp. 656, 657.

[39] Alexander Dalrymple, *Collection of Voyages* (London, 1775); "A Journal of a Voyage in His Majesty's Ship, Paramore Pink, Edmond Halley Commander, Anno 1699 and 1700."

[40] Ole Rømer, *Adversaria* (Copenhagen, 1910), p. 214.

[41] Dorsey, see reference 2, pp. 366–367, quotation from Fahrenheit's letter.

[42] Rømer, see reference 40, pp. 202, 207, 210–211, 213. Cf. note 2 above.

[43] Trans. Roy. Soc. (London) **26**, no. 324, (1709).

[44] *Phil. Trans. . . . Abr.*, reference 27, Vol. 5, p. 98.

[45] Francis Hauksbee, *Physico-Mechanical Experiments* (London, 1719), p. 220.

[46] *Phil. Trans. . . . Abr.*, reference 27, Meteorology, esp. Vol. 8, Pt. 2, pp. 468–469, 556, 561, 565, 567, 573, 574, 582, 584, 586, 589, 590, 592–598, 605, 611, 627.

[47] Trans. Roy. Soc. (London) **58**, p. 55 (1768).

[48] Trans. Roy. Soc. (London) **22**, 527–529 (1700). *Phil. Trans. . . . Abr.*, reference 27, Vol. 2, pp. 90–102 (no. 262).

[49] *Medical Essays and Observations by a Society in Edinburgh* (Edinburgh, 1733–1742), Vols. 1–5. George Martine, *Essays Medical and Philosophical* (London, 1740), p. 230.

[50] Dorsey, see reference 2, p. 366.

[51] Meyer, reference 2, Nature **82**, 298 (1910). Van Swinden, reference 12, pp. 59, 61. *Phil. Trans. . . . Abr.*, reference 27, Vol. 8, Pt. 2, p. 570.

[52] Van Swinden, reference 12, pp. 64, 65.

[53] Dorsey, reference 2, p. 366.

[54] Van Swinden, reference 12, p. 65.

[55] Dorsey, reference 2, p. 367. *Phil. Trans. . . . Abr.*, reference 27, Vol. 10, Pt. 2, p. 429.

[56] Van Swinden, reference 12, pp. 209–211.

[57] Louis Cotte, *Mémoires sur la Météorologie* (Paris, 1788), Table II. See also Van Swinden, reference 12, p. 154, for a slightly different reconstruction.

[58] Van Swinden, reference 12, p. 86. *Phil. Trans. . . . Abr.*, reference 27, Vol. 10, Pt. 2, p. 438.

[59] Van Swinden, reference 12, *passim.*

[60] Cotte, reference 12, p. 130.

[61] *Phil. Trans. . . . Abr.*, reference 27, Vol. 9, p. 478.

[62] Huygens, reference 3, Vol. 5, pp. 188, 228. Gunther, reference 1, Vol. 6, p. 24; Vol. 7, p. 651.

[63] Van Swinden, reference 12, p. 230.

[64] Van Swinden, reference 12, p. 117. *Histoire de l'Académie Royal* (for 1765) (Paris, 1768), p. 219.

[65] De Luc, reference 24, Vol. 1, p. 343.

[66] Van Swinden, reference 12, pp. 237–238.

[67] Trans. Roy. Soc. (London) **22**, no. 270, pp. 824–829. For an English translation, see William Francis Magie, *Source Book in Physics* (McGraw-Hill Book Company, Inc., New York and London, 1935), pp. 125–128.

National Science Foundation Fellowships

The National Science Foundation offers some hundreds of predoctoral and postdoctoral graduate fellowships for advanced study and research in the Biological, Engineering, Mathematical, Medical, and Physical Sciences, for the year 1952–1953. Selection of persons for fellowships will be made from among citizens of the United States solely on the basis of ability.

Basic stipends range from $1400 to $3000 per year. Appointments are for one year. The closing date for applications is January 7, 1952, and awards will be announced about April 1, 1952. The Foundation requires that each applicant for a predoctoral fellowship shall take a Fellowship Record Examination. Applications will be evaluated by the National Research Council.

Further information and application blanks may be obtained from the offices of Graduate Schools throughout the United States, or directly from The Fellowship Office, National Research Council, 2101 Constitution Avenue, Washington 25, D. C.

Note on Electromagnetic Moment of Inertia

W. R. Dixon
National Research Council of Canada, Ottawa, Canada
(Received April 9, 1951)

It is shown that the angular momentum of the stationary electromagnetic field surrounding a classical charged particle may be considered to be localized in the charge distribution. When the magnetic field is considered to be due to rotation of the charge, there exists a moment of inertia tensor relating the angular momentum of the field and the energy of the magnetic field to the angular velocity, just as the angular momentum, kinetic energy of rotation, and angular velocity are related in ordinary mechanics.

THE concept of the electromagnetic mass of an electron is usually illustrated in physics courses by a calculation of the electromagnetic momentum—carried by a spherically symmetric electron moving with constant velocity, or by a calculation of the energy of its magnetic field. One obtains the well-known result that $m=4U/3c^2$, where U is the electrostatic energy, and c is the velocity of light.

It is perhaps not generally realized that an electromagnetic moment of inertia may be defined in an analogous way. If we consider a charged particle rotating with constant angular velocity $\boldsymbol{\omega}$, and confine our attention to the time independent fields $\mathbf{E}$ and $\mathbf{H}$, it is found that the angular momentum of the fields $\boldsymbol{\theta}$, and the energy of the magnetic field T, are given by

$$\boldsymbol{\theta}=\boldsymbol{\Phi}\cdot\boldsymbol{\omega} \tag{1}$$

and

$$T=\tfrac{1}{2}\boldsymbol{\omega}\cdot\boldsymbol{\Phi}\cdot\boldsymbol{\omega}, \tag{2}$$

where $\boldsymbol{\Phi}$ plays the role of a moment of inertia tensor. The elements of $\boldsymbol{\Phi}$ are given by

$$(\varphi_{ij})=\frac{1}{c^2}\int\int\frac{\rho(\mathbf{r})\rho(\mathbf{r}')(\mathbf{r}\cdot\mathbf{r}'\delta_{ij}-x_i'x_j)}{|\mathbf{r}-\mathbf{r}'|}dv'dv, \tag{3}$$

where ρ is the volume charge density, $\mathbf{r}$ is a position vector with components x_1, x_2, and x_3, dv is an element of volume, and δ_{ij} is the Kronecker delta symbol. It will be noted that $\boldsymbol{\Phi}$ is symmetric.

DERIVATION OF RESULTS

In order to demonstrate Eq. (1) it is first necessary to show that the angular momentum of the electromagnetic field may be considered to be localized in the charge distribution. The angular momentum is defined by the integral

$$\boldsymbol{\theta}=(1/4\pi c)\int\{\mathbf{r}\times(\mathbf{E}\times\mathbf{H})\}dv,$$

which may be transformed into

$$\boldsymbol{\theta}=(1/c)\int(\mathbf{r}\times\mathbf{A})\rho dv, \tag{4}$$

where $\mathbf{A}$ is the vector potential. This transformation depends only on the relations

$$\begin{aligned}&\mathrm{div}\mathbf{E}=4\pi\rho, &&\mathrm{curl}\mathbf{E}=0,\\ &\mathrm{div}\mathbf{A}=0, &&\mathrm{curl}\mathbf{A}=\mathbf{H},\end{aligned}$$

and the conditions that $|\mathbf{E}|\sim1/r^2$ and $|\mathbf{A}|\sim1/r^2$ for large r. These conditions imply that at large distances the fields $\mathbf{E}$ and $\mathbf{H}$ behave as if they had their sources in a point charge and magnetic dipole, respectively. The steps in the transformation of the integral are as follows:[1]

(*i*) $\mathbf{E}\times\mathrm{curl}\mathbf{A}=\mathrm{grad}(\mathbf{E}\cdot\mathbf{A})-(\mathbf{E}\cdot\nabla)\mathbf{A}-(\mathbf{A}\cdot\nabla)\mathbf{E}$, where a term with curl$\mathbf{E}$ vanishes.

(*ii*) $\mathbf{r}\times(\mathbf{E}\times\mathrm{curl}\mathbf{A})=-\mathrm{curl}(\mathbf{E}\cdot\mathbf{A}\,\mathbf{r})-\mathbf{r}\times[(\mathbf{E}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathbf{E}]$.

(*iii*) The integral of curl$(\mathbf{E}\cdot\mathbf{A}\,\mathbf{r})$ may be transformed into a surface integral which vanishes when taken at infinity.

(*iv*) $\mathbf{r}\times[(\mathbf{E}\cdot\nabla)\mathbf{A}+(\mathbf{A}\cdot\nabla)\mathbf{E}]=(\mathbf{E}\cdot\nabla)(\mathbf{r}\times\mathbf{A})+(\mathbf{A}\cdot\nabla)(\mathbf{r}\times\mathbf{E})$.

[1] The vector identities used in this transformation may be found, for the most part, in Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1941), p. 604.

(*v*) Use the theorem that

$$\int (\mathbf{F}\cdot\nabla)\mathbf{G}dv = -\int \mathbf{G}\,\mathrm{div}\mathbf{F}dv,$$

where the integrals are taken over all space, and where **F** and **G** are two vectors which vary in magnitude as $1/r$ and $1/r^2$ for large r.

(*vi*) Use the relations $\mathrm{div}\mathbf{A}=0$ and $\mathrm{div}\mathbf{E}=4\pi\rho$ in the preceding step. This gives Eq. (4).

Consider now a rotating model of a particle with a current density given by

$$\mathbf{J}=\rho(\boldsymbol{\omega}\times\mathbf{r}), \tag{5}$$

and thus the vector potential given by

$$\mathbf{A}(\mathbf{r})=(1/c)\int(\mathbf{J}(\mathbf{r}')/|\mathbf{r}-\mathbf{r}'|)dv'. \tag{6}$$

The use of Eqs. (5) and (6) with Eq. (4) leads to Eqs. (1) and (3).

The magnetic energy is given by

$$T=(1/8\pi)\int \mathbf{H}^2dv$$

$$=(1/2c)\int \mathbf{J}\cdot\mathbf{A}dv. \tag{7}$$

The use of Eqs. (5) and (6) with (7) leads to Eq. (2).

If ρ has spherical symmetry, it is easily shown that $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ point in the same direction, so that a scalar definition of the electromagnetic moment of inertia is afforded.

DISCUSSION OF RESULTS

It is not to be expected that a classical model of a rotating particle could account for the observed spin and magnetic moment of the elementary particles. For example it is easily shown that if an electron of radius of the order of e^2/mc^2 is to possess an angular momentum of the order of $\hbar$, the outer parts of the electron must have velocities well in excess of the velocity of light. It has been suggested by Kaempffer[2] that this difficulty may be circumvented by simply associating with the electronic charge a magnetic dipole of the experimentally observed strength.

An interesting extension of these calculations of the angular momentum is to suppose that the mass of the particle is also entirely electromagnetic in origin, and to calculate the ratio of the magnetic moment of a rotating charge distribution to the angular momentum in terms of e/mc. It is found that for a spherical shell model this ratio is exactly e/mc, while for other distributions the value is greater than e/mc. It does not appear, however, that any particular significance can be attached to these results.

[2] F. A. Kaempffer, Can. J. Research **A28**, 336 (1950).

Taylor Memorial Laboratory Manual

The committee of the Association charged with the preparation of a manual of advanced undergraduate laboratory experiments in physics as a memorial to the late L. W. Taylor is anxious to receive further suggestions of experiments to be included in the volume. The experiments should be suitable for use in undergraduate laboratory courses beyond the usual beginning course.

A number of good experiments have already been contributed, but more are wanted. Descriptions need not be elaborate, and they can omit the theoretical background unless the experiment is unusual. Standard experiments as well as unfamiliar ones will be included in the manual. Descriptions should include details of construction of the apparatus if it is not standard, the best method of using it, sources of possible error, and the accuracy of the results to be expected.

All suggestions, whether finally included in the form in which they are submitted or after modification, will be given full consideration and careful evaluation. Contributions should be sent to Joseph D. Elder, Harvard University Press, 44 Francis Avenue, Cambridge 38, Massachusetts.

On the Analysis of Transfer of Training*

LEWIS A. DEXTER

AND

ROBERT A. THORNTON, *Brandeis University, Waltham, Massachusetts*

(Received March 12, 1951)

If what is learned is to be transferable, it must be done in terms of generalized intellectual processes. These processes must be explicitly identified and taught in such a way as to permit students to generalize what is particular and to apply what is general. The selection of curricular material must be done in terms of these processes rather than in terms of a traditional body of facts. Physics, because of its postulational-deductive character, offers an excellent opportunity for this conception of education. Illustrations from physics and social science are given. They indicate the possibility of transfer in terms of common rational processes within and across customary subject-matter areas. If any significant transfer is to be achieved, the following are necessary: less emphasis on memorization of unconnected textbook propositions and formulations which permit emphasis on conceptual features. This cannot be done unless teachers of physics give scholarly attention to the methodology of physics and to the methodology of certain educational problems. This approach not only has educational advantages, but entails advantages for a greater mastery and understanding of physics itself.

THE current emphasis on general education involves the premise that learning is transferable. There has been no satisfactory analysis of the conception of transfer. What do we mean by transfer? What is transferred? And the like. The present paper represents an attempt to clarify this problem.

The test of transfer evidently is: Can a skill or technique learned in one field, subject, or area be readily applied to another? Our problem then becomes one of defining the intellectual habits, and the conceptual skills and techniques, that do, in fact, permit transfer.

We are in agreement with the view expressed in an excellent paper by Orata.[1] He shows, among other things, the importance of the intellectual processes such as logical thinking in the activities of learning and of transfer. These processes are central in the acts of creation, construction, building up attitudes, and idealization. He says, further, that the bulk of evidence supports the belief that transfer takes place chiefly in terms of a consciously generalized intellectual technique.

We start with three tentative assumptions: (1) It is possible to define, describe, and teach transferable intellectual habits. (2) If such transfer were a main concern of teachers in basic science courses, from 25 to 50 percent of the time now spent in repeating introductory assumptions could be saved; hence, in the same curricular period much more could be accomplished. For instance, if students learn to recognize generalizations and to think about the nature of evidence in *any* course, they can continually apply these skills elsewhere. (3) The content of a curriculum should be selected in terms of method rather than in terms of facts or tradition. This hypothesis has a long and respectable history, but we know of no series of investigations which bear directly on it. Once this hypothesis is taken as a basis for procedure, the problem evidently becomes: How is method best taught?

Within the domain of physics, the structure of physical theory in terms of its postulational-deductive character offers excellent opportunity for effecting transfer. Because of this, the good physics teacher consciously or unconsciously teaches transfer within the universe of discourse of physics. Many physics teachers, however, effect transfer within physics without having an

* This article is rewritten with modifications, additions, and some deletions from Lewis A. Dexter's "On teaching the systematic transfer of training," (Harvard Educational Rev. 19, 127–141 (1949), by permission), which was in turn based on papers read by Robert A. Thornton and by Lewis Dexter at the Seventh Annual Conference for Teachers of the Social Sciences in Secondary Schools and Junior Colleges at the University of Chicago, July 22, 1947.

[1] Pedro T. Orata, Educational Administration and Supervision 21, 241–264 (1935); Am. J. Phys. 4, 149–152 (1936) (digest).

explicit verbalized knowledge of the methods they employ; consequently, they are not likely to be skillful in effecting transfer to other fields. To a very much lesser degree the teachers of other theoretical subjects, such as classical economics, teach transfer in their own domain.

It must be emphasized that transfer will not be achieved as the automatic result of preaching transfer. The student must learn to recognize that methods of analysis which he has learned somewhere else apply to a new situation, often one which seems quite different. In order to instruct the student along these lines, it will be necessary to identify explicitly the basic aspects of the scientific method of the discipline which they are learning. *In order to achieve such explicit identification, teachers will frequently find themselves forced to reformulate many of these aspects,* particularly since existing textbooks are not organized *primarily* to teach transfer. This approach is deducible from a widely accepted proposition of the psychology of learning: "For a general principle to transfer, it must usually be learned as one applicable to more than one situation. Otherwise it is unlikely to transfer· · ·. A general principle or any general relations must be learned *as general,* as reaching beyond the specific field in which they are learned."[2]

I

We contend that all problems in general education must be viewed in terms of their relevance to the experimental social psychology of the transfer of learning. We believe that many such problems can profitably be analyzed in terms of the viewpoints developed by the newer sciences of communications. Because of the frequent abuse of the notion of formal discipline as a means of transfer of training, it is perhaps desirable to emphasize that our position is that transfer takes place only where there are common intellectual habits and is much more likely to occur when there is an effort to teach transfer. The present reaction against transfer of learning is in our judgment absurdly overgeneralized and, if it were applied consistently, would involve denial of the possibility of any significant general education.

There are many intellectual habits that can be taught equally well in the natural or social sciences; "definition" is a case in point. For instance, a student who becomes systematically aware of the use of definitions as instruments of abstraction in *any* one science will find it very much easier to "pick up" the basic notions of any other science, because the process of definition is basic to all sciences.

For example, operational definitions are emphasized by many physicists, sociologists, and psychologists. The use of operational definitions sometimes involves certain epistemological assumptions; it always involves certain methodological ones. Without wishing, in this article, to take any position as to the merits or demerits of any of these assumptions, it is clear that a student will not know when he can use operational definitions fruitfully unless he has some idea as to the nature of the assumptions he is making in using them. Additionally, he is far more likely to be able to use operational (or any other) definitions satisfactorily for his purposes if he is aware of the possibilities of using other types of definition. A student who has learned to think about definitions as instruments of analysis is likely to be able to apply this method of thinking to a wide variety of situations; he will have an advantage over anyone who has a narrower and sometimes erroneous conception of definition.

We do not claim, of course, as the foregoing example will show, that anything completely new is suggested here. Good teachers from the Greeks on down have unquestionably taught students about definition in the manner just recommended. What we do claim is that we are generalizing an aspect of the performance of the best teachers[3] which has made them the best teachers; in other words, we believe that we are contributing to making the art of good teaching more nearly communicable.

Intellectual habits vary enormously in their seeming difficulty. Such abilities as multiplication, division, subtraction, and addition are usually regarded in our culture as quite easy, probably because they are taught mechanically; that is, very often no understanding of the

[2] Boring, Langfeld, and Weld, *Introduction to Psychology* (John Wiley and Sons, Inc., New York, 1939), pp. 335–6.

[3] "Best" in the intellectual sense, of course, disregarding rhetorical advantages which one teacher has over another.

basic intellectual processes involved is communicated. At some levels, and for some purposes, mechanical learning is all that is needed and desired. But the more closely one comes to the ideal of liberal education, the more one presumably wishes to stress understanding, meaning thereby ability to explain *why* a principle works as well as *how* it works.

The pragmatic justification for learning "why" is that if one knows why, then one is better equipped to deal with new, unforeseen, changed, emergent circumstances. The student who "fudges" algebra ordinarily cannot apply the processes he has memorized to new situations; often he does not even perceive that the new situations call for algebraic analysis. And so in college many students of statistics, chemistry, and sociology learn a good deal about the formulas for sampling; yet they learn nothing of the logic of sampling and hence make egregious errors in evaluating newspaper reports or medical advice.

Whitehead has made our point for us: "We shall ruin mathematical education if we use it merely to impress general truths. The general ideas are the means of connecting particular results. After all it is the concrete special cases which are important. Thus in the handling of mathematics in your results you cannot be too concrete and in your methods you cannot be too general. The essential course of reasoning is to generalize what is particular and then to particularize what is general. Without generality there is no reasoning, without concreteness there is no importance."[4]

This approach can be applied to all subjects with theoretical aspects. In our own teaching we have stressed, far more than most teachers, specific techniques of leading students to generalize and then to apply, to apply and then to generalize, continually seeking new examples, constantly urging and requiring students to find experience outside the textbooks and the classroom. This effort to teach generalization and application has forced us to adopt a number of unusual techniques in regard to examining, assignments, and note-taking; and the development of the educational theory we are here presenting can best be undertaken by systematic attempts at further innovation in these areas of pedagogy.

Another example of a general skill that can profitably be taught in almost all courses is the ability to ask oneself, "What biases and preconceptions of my own are likely to lead me to make an inaccurate or unsatisfactory estimate of this particular situation which I am analyzing?" Harold Lasswell[5] has shown the importance of this skill for the study of questions of public policy and has suggested techniques for developing it, but its implications are broader. If one learns, for example, that he is prone to identify himself with the minority group or cause in any situation, and then attempts to rationalize his sympathies, this awareness may be of utility in analyzing his scientific as well as political biases. A person may as a result of irrational biases espouse or reject Rhine's mental telepathy theories without adequate analysis of the evidence for them. The teacher of physical science might very well show how new scientific models and patterns of explanation have been rejected on irrelevant and unreasonable grounds, essentially because of the biases prevalent in the scientific world of the day;[6] hence, he might equip his students better to accept the unorthodox but valid scientific ideas of our day; and incidentally might also help train them in objectivity as to their civic interests.

One of the most general skills with which teachers of all subjects are concerned is *explanation*. The demand for an explanation of a situation or fact is synonymous with the demand that it be made intelligible. Certainly, any scientific method is a means whereby intelligibility is sought. One of the most frequently used types of explanation in the process of teaching is explanation by analogy. Very often this is at the common sense level. The identification of similarity is the beginning of explanation. The use of models by scientists is certainly an extension of explanation by analogy. In this case, it takes on a more formal character and is usually more abstract. This type of explanation is given in con-

[4] A. N. Whitehead, *The Aims of Education* (Mentor edition, New York, *ca* 1948), p. 63.

[5] H. D. Lasswell, *Democracy Through Public Opinion* (George Banta Publishing Company, Menasha, Wisconsin, 1941), Chapter 4, "Know thyself."

[6] A group of consultants to the Office of Naval Research (Barton Morgan, *Chairman*, E. Lindquist, J. D. Russell, R. A. Seaton, J. Van Pelt, and L. A. Dexter) prepared a duplicated memorandum *On the In-Service Training of Naval Natural Scientists* (SP: 48–7: RP, January 31, 1948), Secs. IV and V of which deal with the cultivation of ability to think and of to accept original ideas.

nection with scientific theory and takes the form of deduction.

Awareness of the possible fallibility of any particular model or pattern of analogy should certainly be established. The mental models used in modern physics offer excellent opportunity for doing this. Dirac says, "Her [Nature's] fundamental laws do not govern the world as it appears in our mental picture in any very direct way, but instead they control a substratum of which we cannot form a mental picture without irrelevancies."[7]

So, also, there is a tendency to interpret behavior in one society in terms of categories of analysis developed in another. This form of explanation by analogy interferes with effective prediction and control; the simplest and least controversial instance of this is probably the tendency of American college students to assume that the motivations which they can imagine are the only motivations in the world and to regard any other motivations as the clear result of abnormality. Indeed, the teacher of literature often is distressed because of the inability of his students to follow the motivational premises of the characters of Marlowe or Trollope or Tennyson.

II

The often quoted Eddingtonian example of the physics examiner who is interested in the general properties of masses rather than in the particular weights of elephants will serve to illustrate our objective. To be sure, the examination question asks about an elephant sliding down a hill; but as the problem is solved, the peculiar characteristics which differentiate an elephant from any other object of similar mass fade out. However, from a pedagogic standpoint, the illustrations needs to be analyzed beyond the point to which it is customarily taken. The reason why the examiner states the problem in terms of an elephant is presumably because he desires to find out whether the student can recognize how to apply general analytic conceptions and techniques to specific situations. Ability to calculate glibly in terms of abstract masses is, in fact, for most situations of little utility, *unless* the student is able to work out these calculations in regard to particular concrete things, ships, bridges, elephants, houses, et cetera. In some cases, students acquire skill at abstract manipulation of symbols without any corresponding ability to utilize the symbols when faced by particular problems.

Eddington's point has often been misunderstood. Since the emphasis becomes one of teaching intellectual habits, rather than facts, some people feel consequently that facts become unimportant. One of us once visited a class given by a prominent progressive educator, while the latter was absent; in attempting to refute some of the applications of philosophical thought to contemporary politics in which the class was indulging, one of the students quoted certain facts. He was immediately brushed off by other students who remarked: "Mr. X says 'facts are not important. We shouldn't worry about facts.' "

Certainly students should learn a great deal about facts; first, of course, they should learn that facts are important. They should learn many skills in regard to the discovery and verification of fact-statements. But if there is to be appreciable transfer of these skills, their classroom experience should not be focused upon the learning of facts as ends in themselves. It should be focused upon determining what facts are relevant, where to ascertain them, and how to test allegations of fact.

In some subjects, typically American government, and introductory anatomy, students learn facts and nothing but facts. In other subjects, statistics and introductory chemistry, students learn general rules, but are likely to be lost when confronted with concrete problems. The educational process should focus on relating general intellectual habits to the analyses of particular situations.

Let us now return to the question raised at the end of our third assumption: How is method best taught? We can begin with an emphatic negative. It is not best taught by any technique that encourages students to learn propositions by heart without learning their meaning in use and application. Nevertheless, the pressure of the American educational system has been towards the debasement of learning into the memoriza-

[7] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Oxford University Press, New York, 1935), second edition, p. 7.

tion of a set of unconnected propositions, suitable for purposes of objective testing.

The fact that we are confronted with students who have been brought up within an educational system that stresses the *uncritical* learning of propositions creates many difficulties when we try to teach method. Students who have developed the habit of thinking of all propositions as though they are fact propositions, whether they be opinions, values, preferences, hypotheses, theorems, tautologies, inferences, or facts, expect to be asked to handle methodological prescriptions in the same way. Unless the teacher uses a technique of classroom teaching which requires students to *apply* the newly learned conceptions and skills to unfamiliar situations—that is, to *practice* using them as instruments of analysis—he is likely to find them misinterpreting entirely his stress on method. A teacher interested in method must therefore think carefully about his technique of examining, since examinations and grades are regarded by most students as clues to what the teacher really wants him to do.[8]

It is appropriate to say that, for a long time, physics teachers have made more use of this technique than teachers of other fields of knowledge. This has been done by an emphasis on problem-solving involving new and novel situations which require simultaneous solutions rather than mere substitution in formulas.

III

We shall now elaborate our discussion of transferable skills.[9] A skill which is of the greatest importance in scientific activity and in daily life is what may be called *systematic questioning of evidence*, that is, the raising of questions about the nature and validity of evidence whenever a problem is encountered. A warning is in order: some students assume that once the evidence is accepted, a fact is to be accepted as true once and for all. The skill which we are trying to identify would be better indicated by the question: What is the present state of the evidence or what does the current evidence suggest? Those who are skillful in this will be prepared to insist that the alleged evidence shall be circumscribed according to time, place, and source.

The students will have learned to ask themselves, before making any important decisions: What facts are we taking as basic? Does the evidence need to be re-examined in light of new knowledge or observations? There is nothing new about suggesting that these matters are important. However, it is not common for teachers in introductory general courses to deal with the theory and limitations of evidence as such.[10] One of the surest ways for an instructor to exasperate textbook-trained students is to ask them: "How do you know?" It rarely occurs to them that authority is not the only type of evidence. Consequently, such students experience great difficulty when forced to analyze concrete situations in terms of textbook propositions, because, among other things, they do not know how to determine when a proposition is applicable. On the other hand, the better students, when provided with some clue, have little trouble in deducing the relevant logical propositions.

Systematic analysis of the relevance and pertinence of evidence would demand among other things a self-conscious examination of the nature of extrapolation and the recognition and analysis of the validity of analogies. These are clearly key problems of all science and all practical experience. Few students, unfortunately, acquire any realization of their existence.

Courses in both physical and social science would be more valuable if this subject were treated from the viewpoint developed by literary critics such as Kenneth Burke and I. A. Richards.[11]

[8] See L. A. Dexter, School Rev. **55**, 534–41 (1947).

[9] This section is based on L. A. Dexter, Am. Sociol. Rev. **11**, 146–50 (1946).

[10] Hence, we find the situation described by J. W. N. Sullivan, *Aspects of Science* (Smith and Cape, London, 1923), "On learning science," pp. 85–87. "It is a well known fact that a really intelligent child finds great difficulty in believing that the earth is round. Stupid children, on the other hand, believe anything they are told. The difficulty experienced by the first child is due to the fact that, in however elementary a way, it is conscious of the implications of the statement···. Most textbooks [however] are designed for those who can swallow statements at great speed···. I have known students able to solve very difficult problems [by use of textbook propositions] and yet entirely unable to meet, in any way, a skeptical attack upon the fundamental theorem [involved]."

[11] K. Burke, *Attitudes Toward History* (New Republic, New York, 1938), 2 Vols., and I. A. Richards, *Interpretation in Teaching* (Harcourt, Brace and Company, Inc., New

Richards, in discussing clarification and the crystallization of indeterminate and indefinite ideas into a distinct systematic structure of ordered thought, points out that we necessarily find ourselves employing devices of *metaphor* and *parallel* or introducing operations of thought to be treated by analogy with quite other matters. "And," he adds, "however we try to avoid the seemingly awkward, unsatisfactory and regrettable 'accident,' we fail. We substitute another metaphor—recognized as such or met—for the one we are eluding. There is no escape. What we are attempting to dispense with is no accident but the essence of thinking. *Thinking is radically metaphoric.* Linkage by analogy is its constituent law or principle, its causal nexus···." Once aware of this point a teacher can sharpen his students' conception of the instrumental nature of ideas and theories, wean students away from the naive assumption that science is a dogma, and enable them to realize that many apparent problems about reality and absolute validity have no genuine significance.

Skill in looking for the basic metaphors of a theory, learned in any discipline, can be transferred to any other; it might well be classed as the skill of epistemological analysis. As Burke has indicated, the universe is like a cheese: it can be sliced any way you please; but the difference between the slicer of cheeses and the critic, scientist, or epistemologist is that the latter can study their methods of slicing and, if it seems wise or expedient, can reslice indefinitely.

Another skill of considerable importance in almost all areas of science and of life is that of statistical analysis and inference, whether it be formal or informal. Statistical techniques may and do vary considerably from field to field. It is not infrequent to hear students in one science complain because "their" statistics is not the same as the statistics of another science. But the logical assumptions determining the nature of statistical manipulations are similar in all fields. These logical assumptions, however, are often not taught. Probability is one such concept—the ordinary student of statistics learns the formulas about such matters as standard deviations mechanically, and consequently is unable to handle situations not treated by the textbook. As a matter of fact, formulas about probable error involve a series of epistemological assumptions that need to be made explicit if their manipulation is to be anything more than a trick. If these assumptions are made explicit and are understood, the student will, other things being equal, be able to interpret and criticize the statistical formulations in a wide variety of fields.

Another skill which is often treated in a more or less mechanical way is randomization; in actual fact, one can randomize effectively only when one is able to select the pertinent characteristics of the universe intelligently. In other words, randomization cannot take place without an awareness of the criteria of relevance. It must be admitted that there is no simple, succinct, easily utilizable statement on how to determine relevance; but it is *a priori* probable that systematic practice in the detection of relevant factors and attempts to formulate what constitutes relevance will develop skills along this line.

The observation of a phenomenon or the presentation of a simple fact, often evokes the question: Why? This is followed by a statement intended to be an explanation of the question. Even though everyone responds with statements designed to play the role of explanation, few people are clear as to the specific characteristics which makes their answers explanations.

The common usage of "explain" suggests any kind of clarification. The clarification may be summarized in terms of how, or who, or where, or what, or when. The purported explanation may be defined in many ways. This range of possibilities of definition of explanation offers a choice for contextual considerations. It is with the determination of this range of possibilities in terms of specific contexts that we are primarily concerned. It is worth noting that diagnosis in medicine is a conception of the same order as explanation in physics or sociology; and that in many areas of human relations, men act upon an implicit diagnostic technique.

Lest it be thought that the skills which we have discussed at some length are regarded by us as *per se* the most important ones, let us mention

York, 1938), pp. 48–49. Italics in quote from Richards are ours. For further discussion of these points see L. A. Dexter's articles on the theories of definition held by Burke and Richards issued (mimeographed) by the Committee on Conceptual Integration, 1940.

a number of other skills which we might have discussed. The choice of the particular skills to emphasize must always to some extent be an arbitrary matter; it will depend upon the values held by staff and administration of the school, the abilities of the staff, and the background of the students. It is clear that limitations of time alone will mean that not all desirable skills can be taught in any given curriculum.

One highly desirable skill is the ability to recognize that instances of a particular phenomenon as defined fall along a continuum rather than that all possess all the characteristics found in the definition. The ability to use language accurately and pertinently, although important enough to merit special courses in report-writing and composition, should also be stressed in the students' other courses. The ability to look for a patterned relationship—to see the interdependence of phenomena—has become more and more significant in terms of recent development in science.

In the development of mechanics, even on the elementary level of instruction, a number of intellectual habits can be generated—habits that can be used throughout all science. For instance, the realization that physical things are very complicated and consequently are very hard to think about in a logically rigorous fashion is certainly an achievement which offers great opportunity for fruitful transfer; it might transfer to the social field.

The important idea of relevance and the process of abstraction may be consciously delineated in mechanics. The student slowly comes to distinguish between properties that are important for the purposes at hand and those that are not. He thus sees the process of abstraction as a process of relevant simplification. For example, the particular conceptualization is shown to depend on the particular aspect of a problem which is of interest on that occasion. The simplest abstraction for the earth when discussing its *orbit* is that of a point-mass. When, however, *ocean tides* or *lunar eclipses* are the topics of interest the earth may be thought of as a rigid sphere. Again, for discussions of *earthquakes*, the earth is thought of as an elastic sphere. If *precession of the equinoxes* is the important interest, the earth is conceptualized as an ellipsoidal rigid body. Aside from developing an appreciation of the use of idealizations such as particles and rigid bodies, the student is given a planned opportunity to develop the habit of looking for relevant mental constructs and data. Further, the role of abstraction, as a recurrent process, is seen in its relation to the possibility of logical structure.

A great many physics teachers have done this sort of teaching without conscious effort within the limits of physical science. It is questionable, however, whether many physicists have transmitted an awareness of the role of relevant simplification to other areas of science and life. It is further improbable that they have given students the opportunity to invent new relevant simplifications.

IV

The question is: How may teachers who are trained in the orthodox manner of specialization get started in the direction indicated in this paper? If, as is frequently the case, they find that emphasis on the value of this approach in terms of its pedagogical value is disregarded or ineffective—or if they remember the undeniable fact that rewards in university life rarely come to those who are interested in teaching as such—it is well to stress other weighty reasons for placing one's emphasis upon mastery of the logical framework of science. Scientists who understand this framework are more likely to make significant scientific syntheses and to grasp or to make new theoretical analyses because such syntheses ordinarily demand a mastery of the logical framework of science.

Another advantage of this approach—one which will appeal particularly to college administrators beset by the controversy about how much laboratory work a student should take—is that it will tend to lead scientists themselves to question the belief that the ordinary laboratory course leads students to develop an understanding of scientific method.[12] In our terms, training in the technical procedures of the laboratory must be accompanied by explicit emphasis upon theoretical and logical understanding; and once such explicit emphasis is undertaken, the basis

[12] On this point, see Lloyd G. Humphreys, J. Gen. Educ. 5, 210–216 (April, 1951).

for the recurrent criticism of required laboratory courses by humanists and majors in the humanities will be removed. A recent paper[13] by four practicing researchers associated with the Bell Telephone Company and three major universities insists that scholarly attention to the logical framework of science is long overdue. They raise the question, for example: "What is the logical framework of organic chemistry? Or equivalently, what are the characteristic ways in which a good organic chemist thinks and works?" They go on to say: "There is no place where a student can learn this directly, no place where it is set forth clearly freed of as many detailed facts as possible."

On the other hand, there is a vast literature dealing with the logical aspects of theoretical physics which could be given initial attention as a starting point for developing interest in these matters. One may select an item from any number of specific topics of physics and be led into the heart of conceptual and relevant epistomological and historical questions. Specifically, an examination of the effect of the special relativity theory on the fundamental concepts of physics offers a rich opportunity for the application of many logical concepts and certain related psychological problems. An appreciation of the meaning of the decline of mechanism in physics could certainly be initiated by careful examination of the literature which deals with the world of experience as an exemplification of mathematical formulae. The possibility of achieving all of the above objectives depends in a large measure on developing generalizations. May we say, in passing, that an insistence upon general intellectual skills by no means implies a vacuous verbalism about method! Method and theory must be understood in terms of applications. Abstract thinking does not exclude concrete thinking.

The suggestions contained in the preceding two paragraphs must be taken as a first approximation. Finally, in order to move in this direction it is essential that a number of science teachers come to think of educational problems as ones to which they bring the same attitude of intellectual honesty as they bring to the sciences. A program of research as a consequence of such an attitude could be instituted. Research in the following areas would be possible: identifying and understanding the logical elements of science, ways and means of making use of these findings in didactic situations, and evaluation of educational results. We suggest as a leading hypothesis for such research the proposition that all assertions about educational method may in principle be made testable and must make use of the theories and findings of social psychology, social anthropology, and related disciplines.

[13] Bode, Mosteller, Tukey, and Winsor, Science **109**, 553–558 (1949).

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. **19**, 481 (1951)].

Barnaby, Bruce E. (*J*), 634 South 18 Avenue, Maywood, Ill.
Derderian, George, 111–36–130 Street, South Ozone Park 20, N. Y.
Hughes, Hartwell M., 649 Athens Boulevard, Los Angeles 44, Calif.
Hutcherson, John Vincent (*J*), 2536 Piedmont Avenue, Apt. 2, Berkeley 4, Calif.
Krausse, Joel Bronaugh, 1815 S. W. High Street, Portland 1, Ore.
Lindsay, Robert, 2139 "R" Street, N. W., Washington, D. C.
Reid, Edward Lewis (*J*), 701 Alamosa, Claremont, Calif.
Stevenson, Donald Thomas, 98 Duff Street, Watertown 72, Mass.
Turnage, Leonard Cheek, Jr., 6027 Hurst Street, New Orleans, La.
Ward, John Noah, Pennsylvania College for Women, Woodland Road, Pittsburgh 6, Pa.
Wright, Kenneth Arthur, 1016 South College, Mt. Pleasant, Mich.
Wuerker, Ralph Frederick (*J*), 237 South Citrus Avenue, Los Angeles 36, Calif.
Zelenka, Raymond Emil (*J*), 1757 Mecca Road, Columbus 11, Ohio.

Laboratory Performance Tests at the University of Minnesota*

C. N. Wall, H. Kruglak, and L. E. H. Trainor
University of Minnesota, Minneapolis 14, Minnesota
(Received August 6, 1951)

The frequent discrepancies between paper-and-pencil grades of science students and their performance in the laboratory point to the need for behavior-sampling tests: Experience with performance tests at the University of Minnesota has shown that their use (1) increases the spread of grades, (2) results in greater motivation, and (3) facilitates the measurement of instructional outcomes. Typical tests in mechanics, electricity, sound, and optics are reproduced and the important phases of the testing program are described. The detailed statistical analysis of the mechanics performance test points to the need of additional experimentation in this field.

I. GENERAL OBSERVATIONS

THE great majority of the substantial general physics courses in this country require some form of laboratory work by the student. This requirement appears to be based on the assumption that a physics course without laboratory work fails to give the student some of the essential elements of physics, elements which presumably can be gotten in no other way. If there are such elements, then surely there is need for tests devised to deal with those elements.

There are two compelling reasons for wanting such tests. In the first place tests furnish the data, and frequently the only data, upon which student achievement is evaluated and grades are determined. The experienced teacher with a small class may be able to make evaluations without the use of tests or examinations. But in a large university where much of the laboratory instruction is in the hands of graduate students, these conditions are seldom satisfied.

In the second place, tests determine to a very large extent what the student studies and how he studies it. Whatever his professed philosophy of education may be, the student is likely to use the principles of William James' pragmatism in deciding on the importance of this or that item in the course. If the tests to which he is subjected exhibit gaps, it is likely that his knowledge of physics will exhibit the same gaps. Thus tests are useful not only in evaluating student achievements but in directing his education.

The traditional test in physics is the written test. Our experience with written tests, whether of the objective or essay type, leads us to conclude that they are not reliable instruments for judging the learning outcomes of laboratory work in physics. Similar conclusions are drawn by Horton[1] in chemistry.

Laboratory outcomes, in part at least, have to do with the abilities and skills to *perform* operations in the laboratory with or without a thorough understanding of the laws and theories *based* upon these operations.

An ordinary Wheatstone bridge is one thing when explained in lecture. It is something quite different when the student is required to measure a resistance with it in the laboratory. Connectors are no longer without resistance, an exact balance point is not obtainable, the slide wire is not uniform, etc. Finally, there may be a broken connector or loose connection which completely vitiates the result.

For these reasons we need performance tests as well as written tests in physics if we are to attach any importance to the learning outcomes of laboratory work. In order to fill this need, the Physics Department of the University of Minnesota some five years ago instituted performance tests as a regular part of laboratory work in all general physics courses requiring such work.

Laboratory work at the University of Minnesota is of the conventional type throughout the year except for the final week of each quarter. Students work in pairs on the experiment at hand. A laboratory manual is used. Reports are written outside of the laboratory period and are

* Based on papers presented at the Iowa Colloquium of College Physicists, June 14, 1951.

[1] R. E. Horton, *Measureable Outcomes of Individual Laboratory Work in High School Chemistry* (Teachers College, Columbia University, New York, 1928).

handed in for grading the following week. In most of the experiments considerable emphasis is placed on error analysis and calculations.

The last week of the quarter is reserved for laboratory performance tests. These tests are two hours in length (one laboratory period). The students work *individually*, not in pairs, in taking these tests.

Each student in a given course takes the same performance test as every other student in the course. Since these tests run for an entire week, it is clear that some students take these at the beginning of the test week and others at the end of the week. Information about the nature of the test gets out and one might expect that those students taking the test late in the week would have a distinct advantage over those taking it earlier. If such advantage exists, it is not reflected in average test grades, for a plot of average test grade per section against the time at which the test is taken reveals no significant change in the average grade. This is a rather amazing result and seems to indicate that laboratory skills cannot be learned without actual manipulation of apparatus. The nature of laboratory performance tests makes it difficult for the student to employ many of the well-known devices for "getting by" in a written test.

The final laboratory grade for each student is a weighted average of his report grades (usually nine) and his performance test grade. The weight of the performance test grade has gradually crept up as our experience with laboratory tests has increased until it is now given a weight equivalent to four reports, i.e., a contribution of about one-third to the final laboratory grade for the quarter.

Before the advent of laboratory tests at the University of Minnesota, it was common knowledge among students and staff alike that laboratory grades had little effect on final course grades. This arose because laboratory grades had little or no spread. They simply raised all grades the same amount. As a consequence, students were not inclined to exert themselves in the laboratory since the results were seldom commensurate with the effort.

The use of performance tests altered this situation. Not only did we begin to get some spread in laboratory grades, but laboratory grades were lowered and began to look more like average lecture quiz grades. For example, the average laboratory grades for two mechanics sections, one before and one after performance tests were instituted, were 78 ± 6 in Fall, 1943 and 70 ± 13 in Fall, 1950. In this particular instance it is see that the spread has been more than doubled and that the average has been lowered by 8 points.

Another important effect of performance tests was the tonic effect which they produced in the laboratory. Many of the students, especially the better ones, enjoy the challenge of a performance test in which they pilot their own ship for awhile without being sprayed with directions and precautions from the laboratory manual or instructor.

Finally it became increasingly evident, as our experience with performance tests broadened, that they could also be used to measure the effectiveness of *different* methods of laboratory instruction.

In a subsequent article a detailed report will be given on the use of laboratory performance tests in comparing two methods of laboratory instruction.

II. THE PROGRAM AT THE UNIVERSITY OF MINNESOTA

In setting up a program of laboratory performance tests one is faced with a technical problem quite aside from the educational aspects of the program. The nature of this technical problem will depend on local circumstances. In a typical quarter at Minnesota, four separate laboratory courses are offered with a total enrollment of eight hundred. Each laboratory section, limited to sixteen students, does one experiment each week in a two-hour laboratory session. Nine experiments are completed per quarter. Laboratory performance tests are given at the tenth meeting of each section. Information pertinent to the organization of the test program is given in Table I. The test program can be divided into three phases: designing and setting up performance tests, administration of the tests, and scoring of the results.

A typical test consists of 8 "short items" and 3 "long items." Since six sections meet at the same time, six testing rooms are required. Each

TABLE I. Organization of test program.

Course	Total no. of sections	Maximum no. of sections meeting at the same time	No. of lab. rooms available
Mechanics I	25	2	
Mechanics II	10	1	3
Electricity and heat	10	2	4
Optics and sound	6	1	2

testing room requires at least 16 test locations to accommodate all the students in a section. This is accomplished by setting up in each test room one rotation of 8 short items and three identical rotations of 3 long items each. At the end of the first hour, students of long-item rotations interchanged with students on short-item rotations. Each test location is numbered, and is provided with a list of the apparatus at that location and a statement of the problem to be done.

Normally one has only one afternoon and evening available to clear out the apparatus from the last regular experiment of the quarter and to set up the six test laboratories, so that considerable planning beforehand is necessary. Many test experiments require special equipment or modifications in the available equipment. The apparatus at each test location is checked for reliability and ruggedness since as many as 160 students use it during the test week. Each experiment is performed beforehand to ensure that sufficient equipment is given, that it operates satisfactorily, and that sufficient time is alloted for the student to complete the experiment. Many small details require care; for example, adjustment screws on some equipment have to be taped over to prevent tampering.

Two typical tests are reproduced below:

University of Minnesota—Physics 5 and 8 Electricity—Spring, 1951

Laboratory Performance Examination

8 short items—6 minutes each; 2 long items—24 minutes each

SHORT ITEMS

1. *Given:* Ac circuit containing R, L, and C components, ac ammeter, ac voltmeter.

   Problem: Find the power consumption in the lamp. (Assume the lamp filament is purely resistive.) REMOVE VOLTMETER LEADS FROM THE AC CIRCUIT WHEN YOU HAVE FINISHED.

2. *Given:* Assorted apparatus numbered 1 to 14.

   Problem: Write down the numbers of those pieces of equipment which you would *need* to determine the electrical equivalent of heat. Do not list equipment that is not absolutely necessary.

3. *Given:* A circuit diagram and the necessary circuit components and connecting wires to assemble this circuit.

   Problem: Using the material available, wire the circuit shown on the diagram.

   HAVE YOUR INSTRUCTOR CHECK YOUR CIRCUIT. THEN DISMANTLE IT BEFORE LEAVING YOUR STATION.

4. *Given:* Dip needle, table of trigonometric functions. The value of the horizontal component of the earth's magnetic field is 0.17 oersted.

   Problem: Find the vertical component of the earth's magnetic field in oersteds. Show your calculations. Hint: First tip circular scale into horizontal position to line up along magnetic meridian.

5. *Given:* *Handbook of Physics and Chemistry*, small scale.

   Problem: Quickly plot the conductivity of aluminum as a function of temperature in the range −160°C to 600°C. Label the axes of your graph.

6. *Given:* An electrical circuit.

   DO NOT OPERATE OR DISTURB THE CIRCUIT.

   Problem: (a) Draw a schematic diagram of this circuit.
   (b) Label all the circuit components. Show the polarity of batteries and other instruments for which polarity is significant in this circuit.

7. PART A.

   Given: Voltmeter and tap key connected across a cell. DO NOT DISTURB CIRCUIT EXCEPT TO OPERATE TAP KEY.

   Problem: Record the voltage across the cell when the circuit is closed.

   PART B.

   Given: Unknown battery connected in series with a milliameter, tap key, and resistance box set at 31 ohms. Note that the resistance of the ammeter is marked on the case.

   OPERATE PUSH BUTTON ONLY.

   Problem: Determine the voltage across battery when circuit is closed. Show your work.

8. PART A.

Given: A Wheatstone bridge circuit. DO NOT DISTURB SETTING OF THE DECADE RESISTANCE BOX.

Problem: What is the resistance of the unknown resistor marked "A"? Show your calculations. Note: To close main circuit, depress tap key.

PART B.

Given: Unknown resistance "B" connected to a dial box Wheatstone bridge.

Problem: Determine the unknown resistance with as much accuracy as possible. Do not change wiring of the circuit. Return the bridge settings to zero when you have finished.

LONG ITEMS

1. *Given:* 0–1 ammeter, 0–3 voltmeter, 0–15 milliammeter, rheostat, storage cell, low resistance #5 (the order of a tenth ohm), high resistance #6 (the order of a few hundred ohms).

Problem: (a) Find the resistance of #5 and show your calculations. Draw a diagram of your circuit and label the components.
(b) Find the resistance of #6 and show your calculations. Draw a diagram of your circuit and label the components.

2. *Given:* Slide wire potentiometer, standard cell, 2 dry cells, table galvanometer, rheostat, switch.

Problem: To construct a direct reading potentiometer, i.e., one in which the position of the balance point in *meters* equals the potential difference being measured. (To measure an unknown potential difference, one would need only to replace the standard cell with the unknown, and find the balance. The unknown potential difference in *volts* would then be given directly by the position of the new balance point in meters.) When you have completed the circuit, call your instructor to check it. Credit will be established on the basis of this check.

University of Minnesota—Physics 6 and 9—Optics and Sound—Spring, 1951

Laboratory Performance Examination

8 short items—6½ minutes each; 2 long items—26 minutes each

SHORT ITEMS

1. PART A.

Given: Telescope and scale.

Problem: Focus cross hairs of telescope on scale. Call instructor to check your focus.

PART B.

Given: Lenses numbered 5, 6, 7, and 8.

Problem: Arrange these lenses (by number) in the order of most converging to most diverging (i.e., strongest positive lens to strongest negative lens).

2. *Given:* Assorted apparatus.

Problem: (a) Name each piece of equipment numbered 1 to 8.
(b) Name one function of each piece of equipment numbered 1 to 8.

3. *Given:* Microscope, transparent block, vernier calipers.

Problem: Find the index of refraction of the transparent block.

4. *Given:* Telescope.

Problem: Find the magnifying power of the telescope. Hint: Focus on a suitable distant object.

5. *Given:* Optical bench, lens, celluloid scale, white screen, graph paper.

Problem: (a) Find the image distances corresponding to about 5 sets of object distances. Make a plot of object distance p against image distance q.
(b) What is the focal length of the lens?

6. *Given:* Resonance tube, tuning fork, meter stick.

Problem: Determine the wavelength in air of the vibration corresponding to the frequency of the tuning fork.

7. *Given:* Concave mirror, vertical wire, optical bench.

Problem: Determine the focal length of the concave mirror.

8. *Given:* Adjusted spectrometer, prism.

Problem: Find the angle of the prism which is opposite that side which is taped. No credit will be given unless all data are carefully recorded.

LONG ITEMS

1. *Given:* Photometer with 2 lamp bulbs, ammeter, voltmeter.

Problem: The "relative efficiency" of a lamp is defined as its light intensity at a fixed distance divided by the power input to the lamp. Using the lamp not marked "standard" as the lamp to be tested, take measurements necessary and plot a graph of "relative efficiency" against "power input." Use as wide a range of power input as possible in plotting the graph.

2. *Given:* Adjusted spectrometer, prism, white light source, mercury lamp, absorbing solution.

Problem: (a) To calibrate the spectrometer.
(b) To find the wavelength of one of the two absorption bands of the solution.

Procedure for Part (a).

Measure the angles of minimum deviation D_m corresponding to the 4 mercury lines whose wavelengths are given below. Then plot a graph of D_m against wavelength using these 4 points.

Procedure for Part (b).

Use the white light as a source, and place the bottle of liquid between source and collimator. Find the angle D_m for the broadest absorption line and determine the corresponding wavelength from the graph of Part (a).

Note:

The absorption lines appear as dark lines across the continuous spectrum. Mercury green line $\lambda=5460$; mercury yellow line $\lambda=5780$; mercury blue line $\lambda=4920$; mercury violet line $\lambda=4360$.

Since 15 to 20 teaching assistants are involved in the administration of the test program, a uniform procedure is necessary. The need for careful supervision is reduced to a minimum so that a student's scores are not biased. Once the tests get under way there is little opportunity to adjust or repair equipment since one section follows upon the heels of another. Thus every precaution is taken to make the tests rugged and to provide quick replacements for easily damaged equipment. Students are briefed before the test begins on the procedure to be followed and on how results are to be recorded. Students are given a warning signal one-half minute before their time expires at each test location, and they change locations on a signal from the instructor. Continual check is made to see that apparatus is functioning as intended. For reasons of fairness and uniformity, each student must find the apparatus at a test location in the same condition as his predecessor found it, e.g., a dial resistance box set at zero ohms. Students' cooperation is necessary and it is forthcoming with good supervision.

To ensure uniform grading by the teaching assistants a key for scoring is provided. This key is sufficiently detailed to minimize possible variations in awarding points. As an example, the key for scoring Long Item No. 1 of the electricity performance test follows.

Maximum score: 20 points for Part (a) and 20 points for Part (b). There are two possible circuit arrangements: (1) In the first the voltmeter is connected across the resistance only, while (2) in the second it is connected across both ammeter and resistance.

Part (a). Resistance 0.25 ohm. Tolerance ± 0.02 ohm. The student must use arrangement (1). If he uses arrangement (2) he will get about 0.50 ohms. Award zero if the student uses arrangement (2). If the student obtains the correct value and shows his calculations, award 15 points. Award 3 points for a correct circuit diagram. Award 2 points for correct labeling—one-half point for each of the 4 components. (Note: A and V are sufficient for ammeter and voltmeter.)

Part (b). Resistance 870 ohms. Tolerance ± 50 ohms. The student must use arrangement (2). If he uses arrangement (1) he will get about 1500 ohms. Award zero if the student uses arrangement (1). If the student obtains the correct value and shows his calculations, award 15 points. Award 3 points for a correct circuit diagram. Award 2 points for correct labeling—one-half point for each of the 4 components.

Note: A rheostat was provided. The student may use it as a rheostat or as a potential divider, or he need not use it at all.

In addition to specific scoring instructions for each individual item, general instructions applying to all items are given, e.g., subtracting points for omission of units or misuse of units.

III. SPECIAL LABORATORY PERFORMANCE TESTS IN MECHANICS

During the fall quarter of 1950 two laboratory performance tests in mechanics were administered to 603 students in elementary physics and

to 18 graduate teaching assistants. The short-item test was designed to measure the understanding of physical principles in terms of apparatus setups, the possession of a few manipulatory skills, and the ability to solve simple problems with instruments and materials commonly found in an elementary mechanics laboratory. The two performance tests are reproduced below.

University of Minnesota—Physics 1a, 4, and 7—Fall, 1950

Short-Item Laboratory Performance Examination in Mechanics

Short items—3 minutes per location

Maximum score = 180

Location No. 1.

Given: Wire gauge, piece of music wire, wire table.

Problem: To find: (a) the number of the wire
(b) the diameter of the wire.

Location No. 2.

Given: Open-tube manometer containing water connected to a gas jet, meter stick, calipers, table of densities.

Problem: Determine the gas pressure at the jet, in centimeters of mercury.

Table of Densities

Air	0.00114	g/cm³
Mercury	13.6	g/cm³
Water	1.00	g/cm³
Glass	2.65	g/cm³

Location No. 3.

Given: Spectrometer with table locked in a given angular position, directions for reading the angular vernier scale.*

Problem: To read the angular setting of the spectrometer table.

Location No. 4.

Given: Box, weights, spring balance, masonite board.

Problem: To find the average coefficient of sliding friction between the box bottom and the masonite surface.

Caution: Zero error differs with position of balance.

Location No. 5.

Given: A tape record of a freely falling body, meter stick. The trace was made by a stylus vibrating 50 times per second.

Problem: (a) To determine the *time* interval between marks *X* and *Y* on the tape.
(b) To determine the average velocity between the marks *X* and *W*.

Location No. 6.

Given: Micrometer calipers, steel shaft.

Problem: To determine the diameter of the shaft.

Location No. 7.

Given: A system of pulleys, weights, meter stick.

Problem: To find the *theoretical* mechanical advantage of the pulley system.

Location No. 8.

Given: Irregular solid, graduate cylinder with water.

Problem: To find the volume of the solid.

Location No. 9.

Given: Meter stick supported at the center of gravity; weights, sliding hooks, unknown weight.

Problem: To find the unknown weight.

Note: Neglect the weight of sliders.

Location No. 10.

Note: There are two parts of the problem, parts *A* and *B*.

A. Given: Equipment labelled *A*. (Jolly balance, weights, dish of water.)

Problem: What additional equipment, if any, is needed to measure the surface tension of water?

B. Given: Equipment labelled *B*. (Inclined plane, steel ball.)

Problem: What additional equipment, if any, is necessary to determine the acceleration of the ball rolling down the plane?

* ANGULAR MEASUREMENT (Typed on a separate card.)

The verniers used on instruments which measure angles are constructed in the same manner as the one described in the Physics Laboratory Manual, p. 160, Par. 2. A common main scale on a spectrometer is one which is marked every third of a degree. The vernier scale may run from minus two divisions up to 20 divisions with each half division marked. With such an arrangement, the vernier scale from zero to 20 covers exactly 13 degrees on the main scale. That is to say, *forty* vernier divisions equal 39 main divisions, so the least count is 1/40th. Since each main scale division is 20 minutes of arc, each vernier mark will then indicate 1/40th of this or ½ minute. Thus the vernier will read directly in minutes and half minutes if it is numbered from zero to 20. The negative two minutes are for convenience. Note that the vernier reading must be added to the reading of its index. That is, if the index shows 65°20′ plus, and the vernier shows 12½ minutes, the measurement is 65°32½′.

Location No. 11.

Given: Vernier calipers.

Problem: To find the closest measurement that can be made accurately for the *English scale* (least count).

Location No. 12.

Given: Wheel and axle, known weight, unknown weight, meter stick, calipers.

Problem: To find the unknown weight.

Location No. 13.

Given: Set *A* of 3 spring balances hooked in series; set *B* of spring balances, two of which are hooked in parallel and the third in series.

Problem:
(a) Find the approximate tension in the string in set *A*.
(b) Determine the approximate reading of the taped balance No. 1 in set *B*.
(c) Determine the reading of the taped balance No. 2 in set *B*.

Location No. 14.

Given: Pieces of measuring equipment numbered 1, 2, and 3† with a definite setting on each. DO NOT DISTURB THE SETTING ON THESE INSTRUMENTS.

Problem:
(a) What quantity is measured by instrument No. 1?
(b) What is the reading on instrument No. 1?
(c) What is the mass of the metal block on instrument No. 2?
(d) What is the length of the glass plate on instrument No. 3?

Location No. 15.

Given: Assorted pieces of equipment lettered *A* to *E*.‡

Problem:
(a) What is apparatus *A* called?
(b) What is apparatus *B* used for?
(c) What is apparatus *C* called?
(d) What is apparatus *D* used for?
(e) For what specific measurement is apparatus *E* designed?

Location No. 16.

Given: Pendulum stand with assorted pendulums, meter stick.

Problem: List the numbers of those pendulums which you would use in an experiment to determine the effect of the bob mass on the period of the pendulum.

Location No. 17.

Given: *Handbook of Physics and Chemistry.* Caution: Do not make any marks in the handbook.

Problem:
(a) Find the value of Young's modulus for nickel.
(b) What is the percentage variation in the value quoted for Young's modulus for nickel?

Location No. 18.

Given: Hare's apparatus with water and unknown liquid, meter stick.

Problem: To find the density of the unknown liquid (colored).

University of Minnesota—Physics 1a, 4, and 7—Fall, 1950

Long-Item Laboratory Performance Examination in Mechanics

Long items—9 minutes per location
Maximum score = 180

Location No. 19.

Given: Irregular solid, container, water, string, balance, and weights.

Problem:
(a) To find the volume of the irregular solid.
(b) To find the density of the material in the irregular solid.

Location No. 20.§

Given: Pendulum, stop watch, electric clock.

Problem: To find the period of the pendulum.

Location No. 21.§

Given: Traveling microscope, ruled grating.

Problem: To measure the distance between successive rulings on the grating. Hint: Measure the distance between two grating lines separated by 5 spaces, then average.

Location No. 22.||

Given: U-tube containing a liquid, meter stick.

Problem: To find the surface tension of the liquid. The *diameter* of the small vertical tube is 0.10 centimeters.

$T = rhdg/2 \cos\theta$.

$\theta = 0°$.

The density of the liquid is 1.00 g/cm³.

† 1. Hydrometer in glycerine.
2. Triple beam balance; weight taped onto pan; system already in balance.
3. Steel rule with glass plate taped on.

‡ *A* = revolution counter; *B* = Jolly balance; *C* = traveling microscope; *D* = cathetometer; *E* = centripetal force apparatus.

§ Experiments 20 and 21 constitute one location.

|| Experiments 22 and 23 constitute one location.

Location No. 23.‖

Given: System of ropes and pulleys, weights.

Problem: To find the *actual* mechanical advantage of the system.

Location No. 24.

Given: Derrick, weight, spring balance in the tie string, and meter stick.

Problem: Find the weight of the boom.

Note: The mass of the suspended weight is marked on it.

Location No. 25.

Given: Semi-analytical balance, set of standard weights, damaged standard weight.

Problem: To find the percentage error in the value stamped on the damaged standard weight.

Location No. 26.

Given: Torsion apparatus, set of weights, torsion rod, graph paper, rule, protractor, calipers.

Problem: (a) To represent graphically the relationship between applied torque and angle of twist of the torsion rod. (Obtain at least 4 points on the graph.)

(b) What conclusions can you draw from the graph?

Note: There is a scale on the side of the torsion wheel.

The items of the two laboratory tests in mechanics were validated as to their importance in meeting the objectives of elementary nontechnical physics. Fifty-nine college physics teachers throughout the country were asked to rate the items on a 4-point scale. There were 32 usable returns. The results of the questionnaire are shown in Table II.

Although there seemed to be a considerable difference of opinion on a number of items with regard to their importance in meeting the objectives of a particular teacher, the majority of the items had a fairly high face validity.

Using the total score on the test as a criterion, the difficulty and the discriminating power of each item were computed for 64 students in the elementary nontechnical physics course (Physics 1a). The results of the item analysis are shown in Table III. The largest number of discriminations between pairs of individuals is made by an item of 50 percent difficulty.[2] In the short-item test there were 18 responses in the 25–75 percent range; there were 8 items above 75 percent and 5 items below 25 percent in difficulty. The long-item test had 7 responses in the 25–75 percent range; there were no items above 75 percent; 2 items were below 25 percent in difficulty. The

TABLE II. Item validation by experts of laboratory performance tests. University of Minnesota—Physics 1a—Fall, 1950.

Item No.	Rating distribution* 1	2	3	4	None
	Short-item mechanics				
1b, 1a	2	6	15	8	1
2	13	11	7	0	1
3	3	10	14	4	1
4	11	15	3	1	2
5a	9	11	8	4	0
5b	18	9	2	3	0
6	8	8	8	4	4
7	7	9	11	3	2
8	13	12	3	2	2
9	20	9	2	0	1
10a	7	12	9	3	1
10b	15	11	5	0	1
11	10	12	7	2	0
12	9	10	7	4	2
13a	15	8	6	2	1
13b, 13c	12	9	8	1	2
14a	9	11	9	3	0
14b, 14c, 14d	11	7	10	2	2
15a, 15b, 15c, 15d, 15e	9	11	9	3	0
16	5	8	9	8	2
17a, 17b	9	8	10	4	1
18	9	7	11	0	5
	Long-item mechanics				
19a, 19b	21	9	1	0	1
20	Question reworded after mailing questionnaire				
21	5	9	10	6	2
22	1	14	11	4	2
23	12	13	6	0	1
24	9	11	9	3	0
25	9	8	9	5	1
26	9	10	7	5	1

* DIRECTIONS FOR RATING TEST ITEMS. How well does this item meet *your* objectives of laboratory instruction in elementary, nontechnical physics? Rating scale:
1. The item meets one of the most essential objectives.
2. The item meets a relatively important objective.
3. The item meets a relatively unimportant objective.
4. The item better be omitted.

[2] D. C. Adkins, *et al.*, *Construction and Analysis of Achievement Tests* (U. S. Government Printing Office, Washington, D. C., 1947), p. 147.

TABLE III. Item analysis of laboratory performance tests. University of Minnesota—Physics 1a—Fall, 1950.

Item No.	A*	B*	C*
	Short-item mechanics $N=64$		
1a	53	28	55
1b	28	37	47
2	35	23	40
3	15	45	36
4	19	36	33
5a	45	33	58
5b	29	48	42
6	21	4	40
7	34	20	43
8	58	29	55
9	70	28	57
10a	86	34	70
10b	78	26	76
11	11	21	28
12	46	44	47
13a	91	20	68
13b	92	21	81
13c	73	8	65
14a	66	8	59
14b	48	11	50
14c	91	31	75
14d	46	32	44
15a	31	0	38
15b	88	8	71
15c	85	3	68
15d	56	33	49
15e	54	21	51
16	10	28	18
17a	88	14	75
17b	46	33	51
18	27	12	41
	Long-item mechanics $N=64$		
19a	73	49	61
19b	65	37	55
20	29	20	38
21	18	16	31
22	33	11	40
23	68	38	58
24	3	10	13
25	50	45	51
26	55	19	53

* A = Difficulty = percent successes; B = Davis discrimination index; C = Davis difficulty index.

Davis discrimination index[3] is a numerical measure of the discriminating power of a test item, based on the top and bottom 27 percent of the total group. A zero discrimination index means that the percentage of successes on the item in the upper 27 percent of the sample is the same as the percentage of successes in the lower 27 percent of the group. A discrimination index of 100 is obtained when everyone in the upper group succeeds on the item, but every one in the lower group fails the item. An examination of Table III shows that the Davis discrimination indices for both laboratory tests were relatively *low* for the sample of students under investigation. The reliability of the short-item test was calculated by the Hoyt method.[4] With $r=0.68$, the test differentiated reliably at the 1 percent level between the 64 selected individuals. The reliability of the long-item test was also 0.68 for the same population sample. The low reliabilities of the two laboratory performance tests could possibly be explained by the homogeneity of the subjects in the sample, by the relatively small number of items in each test, and by the heterogeneity of the test items.

The laboratory tests were graded by the 4 graduate assistants who taught the 64 selected cases and rescored independently by an assistant who had no contacts with the students. The reliability of scoring was determined by calculating the product moment correlations and by using the t-test for correlated measures for comparing the means of the two corresponding sets of grades.[5] The results showed that the correlations were high for all pairs of graders. On the other hand, there were significant differences in the means of five out of ten comparisons. It must be remembered that in spite of a detailed scoring key and oral instructions to the graders, there is a considerable amount of judgment that enters into the scoring of the performance tests. The two mechanics performance tests were administered to five different types of students.

TABLE IV. Average scores on laboratory performance tests. University of Minnesota—Fall Quarter, 1950.

Course Test	Physics 1* Boys; $N=24$	Girls; $N=30$	Total; $N=54$	Physics 1a* $N=174$	Physics 4* $N=94$	Physics 7* $N=281$	G.T.A.* $N=18$
Short-item mechanics	33	24	28	43	44	45	79
Long-item mechanics	36	24	29	42	46	45	83

* Physics 1—General physics—no laboratory required; students took test for extra credit.
Physics 1a—General physics—laboratory required, 11 sections.
Physics 4—Premedical physics, 6 sections.
Physics 7—Engineering physics, 13 sections.
G.T.A.—Graduate teaching assistants.
Very few girls in Physics 1a, 4, and 7; many girls in Physics 1.

[3] F. B. Davis, *Item-Analysis Data* (Harvard University, Cambridge, Massachusetts, 1949).

[4] C. J. Hoyt, Psychometrika 6, 153–160 (June, 1941).

[5] P. O. Johnson, *Statistical Methods in Research* (Prentice-Hall, New York, 1949), p. 75.

The average scores are reproduced in Table IV. It is seen that on the average there were three distinct levels of performance: The lowest score was obtained by the students who were not required to take laboratory for credit; the next category was that of students in the elementary courses with required laboratory; the graduate teaching assistants in physics, as one would expect, were very much better than the sophomore groups. The differences between the scores in the three middle groups are probably not significant. The average score on the short-item test was practically the same as the score on the long-item test for all the groups. The boys in the no-laboratory group did considerably better than the girls.

It is clear that more work must be done with performance tests before definite conclusions can be drawn about their use as evaluation instruments. It is hoped that the above discussion will stimulate experimentation with this type of laboratory tests.

The Output Current of a Nonlinear Device

BERNARD SALZBERG
Naval Research Laboratory, Washington 25, D. C.
(Received May 28, 1951)

A voltage consisting of a cosine term superposed on a steady term is applied to the input of a nonlinear device. The output current of the device is zero for negative voltages, and proportional to the nth power of the voltage when it is positive, the exponent n being 1, $\frac{3}{2}$, or 2. The resultant current may be zero, continuous, or discontinuous, depending on the relative values of the steady term and the peak value of the cosine term. In general, the current may be represented by a Fourier series, the coefficients of the various terms of which are functions of n and the magnitudes of the two components of the voltage. For $n=1$ and $n=2$, these coefficients are expressible in terms of trigonometric functions. For $n=\frac{3}{2}$, the coefficients are given in terms of the complete elliptic integrals K and E. The root-mean-square value of the current, for $n=1$, $\frac{3}{2}$, and 2, is given in terms of trigonometric functions. Graphs of coefficients of the leading Fourier terms, and of the root-mean-square current, are presented for both continuous and discontinuous current, for $n=1$, $\frac{3}{2}$, and 2.

ANALYSES of the behavior of radiofrequency circuits which utilize nonlinear electronic devices as oscillators, amplifiers, or harmonic-multipliers are usually based on results of the following situation: The output current of the nonlinear device is assumed to be zero when the input voltage is negative, and proportional to the nth power of the input voltage when it is positive. The exponent n is 1, $\frac{3}{2}$, or 2, these values having been found typical of many vacuum tubes. The input voltage consists of an alternating term which is a simple harmonic function of time, superposed upon a steady polarizing term. The resultant output current may be zero, may consist of discontinuous pulses, or may be a continuous function of time, depending upon the relative values of the polarizing voltage and the amplitude of the alternating voltage. In general, the output current may be represented by a Fourier series, the coefficients of the various terms of which are functions of the exponent n, the polarizing voltage, and the amplitude of the alternating voltage. These coefficients are essential to analyses of the behavior of circuits utilizing nonlinear devices.[1–4] For $n=1$ and $n=2$, explicit expressions have been derived for the coefficients for the important case of discontinuous output current. For $n=\frac{3}{2}$, coefficients of the leading Fourier terms have been determined by graphical methods for the case of discontinuous output current; explicit expressions for these coefficients, occasionally

[1] F. E. Terman and J. H. Ferns, Proc. Inst. Radio Engrs. **22**, 359–373 (1934).
[2] F. E. Terman and W. C. Roake, Proc. Inst. Radio Engrs. **24**, 620–632 (1936).
[3] W. G. Wagener, Proc. Inst. Radio Engrs. **25**, 47–77 (1937).
[4] F. E. Terman, Trans. Am. Inst. Elec. Engrs. **57**, 640–644 (1938).

required in analysis, do not appear to have been worked out.

In this paper explicit expressions are derived for the coefficients of the Fourier terms when $n=1$, $\frac{3}{2}$, and 2, for the case of continuous output current, as well as for the case of discontinuous output current. When $n=1$ and $n=2$, the coefficients are expressible in terms of trigonometric functions. When $n=\frac{3}{2}$, the coefficients are given in terms of the complete elliptic integrals $K(k)$ and $E(k)$. Explicit expressions are also derived for the corresponding values of root-mean-square output current; these also may be required occasionally. Graphs of coefficients of the leading Fourier terms and of the root-mean-square current are provided for both continuous and discontinuous output current, for $n=1$, $\frac{3}{2}$, and 2.

Aside from its possible practical value in consolidating some old results and providing some new results for which there is also an occasional need, the analysis may be useful for pedagogical purposes as an illustration of the application of Fourier series and elliptic integrals to a current problem of applied physics.

ANALYSIS

The output current i of the nonlinear device, expressed as a function of the input voltage v, is

$$\begin{aligned} i&=0 \quad \text{for} \quad v\leqslant 0 \\ i&=Gv^n \quad \text{for} \quad v\geqslant 0, \end{aligned} \tag{1}$$

where G and n are both constant.

The input voltage is

$$v=V\cos\theta+V_0, \tag{2}$$

where θ is the product of angular frequency and time. The voltage is symmetrical about θ, i.e., it has the same value for $-\theta$ as it has for θ, and it has a period 2π in the typical range $-\pi\leqslant\theta\leqslant\pi$. The current, correspondingly, is also symmetrical about θ, and it also has a period 2π in the typical range $-\pi\leqslant\theta\leqslant\pi$. The form of the current in any other similar range $(2m-1)\pi\leqslant\theta\leqslant(2m+1)\pi$ will be identical with that in the typical range; therefore, only the latter need be considered.

When $V_0\leqslant -V$, the voltage is negative for all values of θ; the current, by Eq. (1), is therefore zero for all values of θ. This is a case of no interest. When $V_0\geqslant V$, the voltage is positive for all values of θ; the current is therefore continuous for all values of θ. In this case it is useful to write Eq. (2) as

$$v=V(\cos\theta+r), \tag{2a}$$

where $r=(V_0/V)$ is a positive parameter having a value greater than or equal to unity. The corresponding form of Eq. (1) is

$$i=GV^n(\cos\theta+r)^n. \tag{1a}$$

When $-V\leqslant V_0\leqslant V$, the voltage is positive for values of θ in the range $-\alpha\leqslant\theta\leqslant\alpha$, where α is defined by $\cos\alpha=-(V_0/V)$, and is negative for all other values of θ. The current is therefore discontinuous, being finite in the range $-\alpha\leqslant\theta\leqslant\alpha$, and zero for all other values of θ. For this case it is convenient to write Eq. (2) as

$$v=V(\cos\theta-\cos\alpha), \tag{2b}$$

where the angle α has a value between 0 and π. The total angle during which the output current is finite, or the total "angle of conduction," is 2α. A useful form of Eq. (1) for this case is

$$\begin{aligned} &i=0 \quad \text{for} \quad -\pi\leqslant\theta\leqslant-\alpha \quad \text{and for} \quad \alpha\leqslant\theta\leqslant\pi \\ &i=GV^n(\cos\theta-\cos\alpha)^n \quad \text{for} \quad -\alpha\leqslant\theta\leqslant\alpha. \end{aligned} \tag{1b}$$

The maximum value of current occurs when $\theta=0$, so that

$$\begin{aligned} I'&=G(V+V_0)^n=GV^n(1+r)^n \\ &=GV^n(1-\cos\alpha)^n. \end{aligned} \tag{3}$$

The mean-square value of the current is obtained by integrating the square of the current over the portion of a period during which it is finite, and dividing by 2π. Using Eq. (1a) this gives, for the case in which the current is continuous for all values of θ,

$$I^2=(G^2V^{2n}/\pi)\int_0^{\pi}(\cos\theta+r)^{2n}d\theta. \tag{4a}$$

The mean-square value of the current, for the case in which the current is discontinuous, is

$$I^2=(G^2V^{2n}/\pi)\int_0^{\alpha}(\cos\theta-\cos\alpha)^{2n}d\theta. \tag{4b}$$

In general, the current may be represented by a Fourier series of the form

$$\begin{aligned} i=I_0&+I_1\cos\theta+I_2\cos2\theta \\ &+\cdots+I_m\cos m\theta+\cdots, \end{aligned} \tag{5}$$

where the coefficients are given by

$$I_0=(G/\pi)\int v^n(\theta)d\theta \tag{6}$$

and

$$I_m=(2G/\pi)\int v^n(\theta)\cos m\theta d\theta. \tag{7}$$

In Eqs. (6) and (7) $m=1, 2, 3, \cdots$, and the integration limits are 0 and π for the case of continuous current, and 0 and α for the case of discontinuous current.

Continuous current, $n=1$.—The maximum value of current is, from Eq. (3),

$$I'=GV(1+r). \tag{3a.1}$$

The root-mean-square value of the current is obtained by putting $n=1$ in Eq. (4a), carrying out the indicated integration, and taking the square root of the result.

$$I=GV(\tfrac{1}{2}+r^2)^{\frac{1}{2}}. \tag{4a.1}$$

The coefficients of the various terms in the Fourier series for the current are obtained by using Eq. (2a) in Eqs. (6) and (7), with the limits 0 and π. The results are

$$I_0=GV_0=GVr \tag{6a.1}$$

$$\begin{aligned} &I_1=GV \\ &I_m=0 \quad \text{for} \quad m=2, 3, 4, \cdots. \end{aligned} \tag{7a.1}$$

Continuous current, $n=2$.—The maximum value of current is

$$I'=GV^2(1+r)^2. \tag{3a.2}$$

The root-mean-square value of the current is

$$I=GV^2(\tfrac{3}{8}+3r^2+r^4)^{\frac{1}{2}}. \tag{4a.2}$$

The coefficients of the terms in the Fourier series for the current are

$$I_0=GV^2(\tfrac{1}{2}+r^2) \tag{6a.2}$$

$$\begin{aligned} &I_1=2GV^2r \\ &I_2=\tfrac{1}{2}GV^2 \\ &I_m=0 \quad \text{for} \quad m=3, 4, 5, \cdots \end{aligned} \tag{7a.2}$$

Continuous current, $n=\frac{3}{2}$.—The maximum value of current is

$$I'=GV^{\frac{3}{2}}(1+r)^{\frac{3}{2}}. \tag{3a.3}$$

The root-mean-square value of the current is

$$I=GV^{\frac{3}{2}}r^{\frac{1}{2}}(r^2+\tfrac{3}{2})^{\frac{1}{2}}. \tag{4a.3}$$

The coefficients of the terms in the Fourier series for the current are given by

$$I_0=(GV^{\frac{3}{2}}/\pi)\int_0^{\pi}(\cos\theta+r)^{\frac{1}{2}}d\theta$$

$$I_m=(2GV^{\frac{3}{2}}/\pi)\int_0^{\pi}(\cos\theta+r)^{\frac{1}{2}}\cos m\theta d\theta.$$

The integrals which appear in these expressions cannot be evaluated in terms of the elementary functions. However, they can be evaluated in terms of the complete elliptic integrals K and E, for which tables are available.[5,6]

The evaluation is carried out as follows:[7] Introduce the variable $x^2=(\cos\theta+r)$; the integral which appears in the expression for I_0 is transformed to

$$2R_4=2\int_{x_1}^{x_0}x^4P^{-\frac{1}{2}}dx,$$

where the limits of integration are given by

$$\begin{aligned} x_0^2&=(r+1) \\ x_1^2&=(r-1) \end{aligned}$$

and

$$P=(1-r^2)+2rx^2-x^4=-(x^2-x_0^2)(x^2-x_1^2).$$

Let $p=1, 2, 3, \cdots$; we note that

$$\frac{d}{dx}(x^pP^{\frac{1}{2}})=\left[px^{p-1}P+\tfrac{1}{2}x^p\frac{dP}{dx}\right]P^{-\frac{1}{2}}.$$

Integration of this expression between the limits x_0 and x_1 yields a recurrence relation between definite integrals of the type R_4, given above; this relation is

$$(p+2)R_{p+3}=2r(p+1)R_{p+1}-(r^2-1)pR_{p-1}, \tag{8a}$$

in which

$$R_p=\int_{x_1}^{x_0}x^pP^{-\frac{1}{2}}dx. \tag{9a}$$

By setting $p=1$ in Eq. (8a), it will be seen that

[5] B. O. Peirce, *A Short Table of Integrals* (Ginn and Company, New York, 1929).

[6] H. B. Dwight, *Tables of Integrals and other Mathematical Data* (Macmillan Company, New York, 1947).

[7] For those interested in following through the details of the evaluation, a useful reference is J. Pierpont, *Functions of a Complex Variable* (Ginn and Company, New York, 1914).

the definite integral R_4 can be expressed in terms of R_2 and R_0. Consequently I_0, which is proportional to R_4, can be written as

$$I_0=(2GV^{\frac{1}{2}}/3\pi)[4rR_2-(r^2-1)R_0].$$

The elliptic integrals R_2 and R_0 can be converted to standard form by means of the transformation $x^2=x_0^2-(x_0^2-x_1^2)\sin^2\phi$, the results being

$$\begin{aligned} R_2&=(r+1)^{\frac{1}{2}}E(k)\\ R_0&=(r+1)^{-\frac{1}{2}}K(k), \end{aligned} \qquad (10a)$$

in which K and E are the complete elliptic integrals of the first and second kinds, respectively, and their modulus is given by $k^2=2/(r+1)$.

The zeroth-order Fourier coefficient is therefore

$$I_0=(2GV^{\frac{1}{2}}/3\pi)(r+1)^{\frac{1}{2}}[4rE-(r-1)K]. \quad (6a.3)$$

The mth-order Fourier coefficients are obtained by means of the same transformations as those used above. The factor $\cos m\theta$ which appears in the integral for these coefficients can be expressed[6] as a sum of terms each of which is proportional to a positive integral power of $\cos\theta$. Since $\cos^n\theta=(x^2-r)^n$, any given coefficient can be written as a sum of terms each of which is proportional to a definite integral of the type R_{2p}. By means of the recurrence relation (8a), the coefficient can be reduced to a form which is linear in R_2 and R_0. Then, by using the conversions to standard form, Eq. (10a), the mth-order coefficient can be expressed as

$$I_m=A_mE(k)+B_mK(k).$$

The results of carrying out this procedure for the leading mth-order coefficients are

$$\begin{aligned} I_1&=(4GV^{\frac{1}{2}}/5\pi)(r+1)^{\frac{1}{2}}\\ &\quad\times[(r^2+3)E-r(r-1)K]\\ I_2&=(4GV^{\frac{1}{2}}/35\pi)(r+1)^{\frac{1}{2}}\\ &\quad\times[4r(2-r^2)E+(r-1)(4r^2-5)K] \quad (7a.3)\\ I_3&=(4GV^{\frac{1}{2}}/315\pi)(r+1)^{\frac{1}{2}}\\ &\quad\times[(32r^4-57r^2+21)E\\ &\quad-r(r-1)(32r^2-33)K]. \end{aligned}$$

Explicit expressions for still higher order coefficients can be obtained, when required, in exactly the same way.

Discontinuous current, $n=1$.—The maximum value of current is, from Eq. (3),

$$I'=GV(1-\cos\alpha). \qquad (3b.1)$$

The root-mean-square value of the current is obtained by putting $n=1$ in Eq. (4b), performing the indicated integration, and taking the square root of the result:

$$I=GV\pi^{-\frac{1}{2}}[\alpha+\tfrac{1}{2}\alpha\cos2\alpha-\tfrac{3}{4}\sin2\alpha]^{\frac{1}{2}}. \quad (4b.1)$$

The Fourier coefficients for the current are obtained by using Eq. (2b) in Eqs. (6) and (7), with the limits 0 and α. The results are

$$I_0=(GV/\pi)(\sin\alpha-\alpha\cos\alpha) \qquad (6b.1)$$

and

$$I_m=\left(\frac{GV}{\pi m}\right)\left[\frac{\sin(m-1)\alpha}{(m-1)}-\frac{\sin(m+1)\alpha}{(m+1)}\right]. \quad (7b.1)$$

Discontinuous current, $n=2$.—The maximum value of current is

$$I'=GV^2(1-\cos\alpha)^2. \qquad (3b.2)$$

The root-mean-square value of the current is

$$\begin{aligned} I=GV^2\pi^{-\frac{1}{2}}[&(9\alpha/4)-(5/3)\sin2\alpha\\ &-(25/96)\sin4\alpha+2\alpha\cos2\alpha\\ &+(\alpha/8)\cos4\alpha]^{\frac{1}{2}}. \end{aligned} \quad (4b.2)$$

The coefficients of the terms in the Fourier series for the current are

$$I_0=(GV^2/\pi)[\alpha+\tfrac{1}{2}\alpha\cos2\alpha-\tfrac{3}{4}\sin2\alpha] \qquad (6b.2)$$

$$\begin{aligned} I_m=(GV^2/\pi)\Big[&\frac{m+2}{m+1}\frac{\sin m\alpha}{m}\\ &+\frac{1}{m(m+1)}\frac{\sin(m+2)\alpha}{(m+2)}+\frac{m-1}{m}\\ &\times\frac{\sin(m-2)\alpha}{(m-2)}-2\frac{\sin(m-1)\alpha}{(m-1)}\cos\alpha\Big]. \end{aligned} \quad (7b.2)$$

Discontinuous current, $n=\frac{3}{2}$.—The maximum value of current is

$$I'=GV^{\frac{3}{2}}(1-\cos\alpha)^{\frac{3}{2}}. \qquad (3b.3)$$

The root-mean-square value of the current is

$$\begin{aligned} I=[GV^{\frac{3}{2}}/2(6\pi)^{\frac{1}{2}}][&27\sin\alpha+11\sin3\alpha\\ &-54\alpha\cos\alpha-6\alpha\cos3\alpha]^{\frac{1}{2}}. \end{aligned} \quad (4b.3)$$

The coefficients of the terms in the Fourier

series for the current are given by

$$I_0=(GV^{\frac{1}{2}}/\pi)\int_0^{\alpha}(\cos\theta-\cos\alpha)^{\frac{1}{2}}d\theta$$

$$I_m=(2GV^{\frac{1}{2}}/\pi)\int_0^{\alpha}(\cos\theta-\cos\alpha)^{\frac{1}{2}}\cos m\theta d\theta.$$

The integrals which appear in these expressions cannot be evaluated in terms of the elementary functions. They can be expressed, however, in terms of the complete elliptic integrals K and E, in very much the same way as was done in the corresponding case of continuous current. Let $x^2=(\cos\theta-r)$, where $r=(V_0/V)=\cos\alpha$, the range of r now being $-1\leqslant r\leqslant 1$. The integral which appears in the expression for I_0 is transformed to

$$2R_4=2\int_0^{x_0}x^4P^{-\frac{1}{2}}dx,$$

where the upper limit of integration is given by $x_0^2=(1-r)=2\sin^2(\alpha/2)$, and

$$P=(1-r^2)-2rx^2-x^4.$$

Let $p=1, 2, 3, \cdots$; we note

$$\frac{d}{dx}(x^pP^{\frac{1}{2}})=\left[px^{p-1}P+\tfrac{1}{2}x^p\frac{dP}{dx}\right]P^{-\frac{1}{2}}.$$

Integration of this expression between the limits x_0 and 0 yields the following recurrence relation between definite integrals of the type R_4, given above:

$$(p+2)R_{p+3}=(1-r^2)pR_{p-1}-2r(p+1)R_{p+1}, \quad (8b)$$

in which

$$R_p=\int_0^{x_0}x^pP^{-\frac{1}{2}}dx. \quad (9b)$$

The definite integral R_4 can be expressed in terms of R_0 and R_2 by setting $p=1$ in Eq. (8b). Thus I_0, which is proportional to R_4, can be written as

$$I_0=(2GV^{\frac{1}{2}}/3\pi)[(1-r^2)R_0-4rR_2].$$

The elliptic integrals R_0 and R_2 can be converted to standard form by use of the transformation $x^2=x_0^2\cos^2\phi$; the results are

$$\begin{aligned}R_0&=2^{-\frac{1}{2}}K(k)\\ R_2&=2^{\frac{1}{2}}E(k)-2^{\frac{1}{2}}(1-k^2)K(k),\end{aligned} \quad (10b)$$

in which K and E represent, as usual, the complete elliptic integrals of the first and second kinds, respectively, and their modulus is given by $k^2=\frac{1}{2}(1-r)=\sin^2(\frac{1}{2}\alpha)$.

The zeroth-order Fourier coefficient is

$$I_0=(2^{\frac{1}{2}}\cdot 4GV^{\frac{1}{2}}/3\pi)\times[(1-k^2)(2-3k^2)K-2(1-2k^2)E]. \quad (6b.3)$$

The mth-order Fourier coefficients are obtained by means of the same transformations as those used above. As in the corresponding case

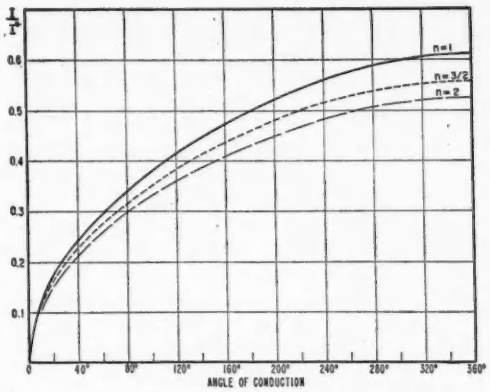

FIG. 1. The ratio of the rms value of output current to the maximum value, as a function of the total angle of conduction, for $n=1$, $\frac{3}{2}$, and 2.

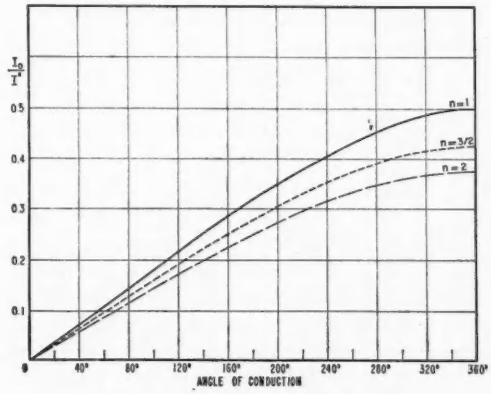

FIG. 2. The ratio of the dc component of output current to the maximum value, as a function of the total angle of conduction, for $n=1$, $\frac{3}{2}$, and 2.

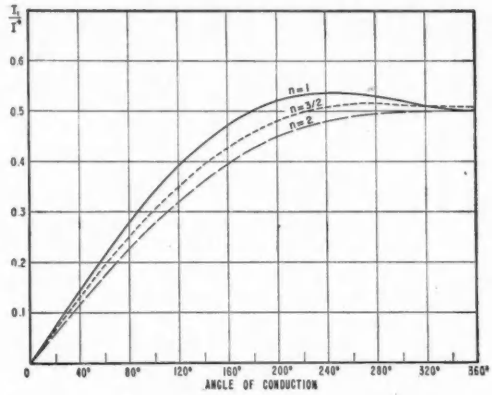

FIG. 3. The ratio of the fundamental component of output current to the maximum value, as a function of the total angle of conduction, for $n=1$, $\frac{3}{2}$, and 2.

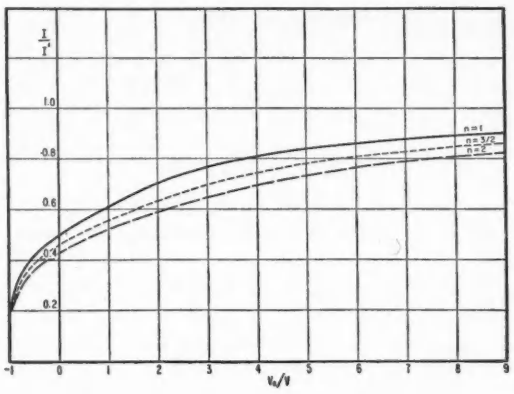

FIG. 5. The ratio of the rms value of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for $n=1$, $\frac{3}{2}$, and 2.

of continuous current, the factor $\cos m\theta$ which appears in the integral for these coefficients can be expressed as a sum of terms each of which is proportional to a positive integral power of $\cos\theta$. Since $\cos^n\theta=(x^2+r)^n$, any given coefficient can be written as a sum of terms each of which is proportional to a definite integral of the type R_{2p}. By use of the recurrence relation (8b), the coefficient can be reduced to a form which is linear in R_0 and R_2. Then, by using the conversions to standard form, Eq. (10b), the mth-order coefficient can be expressed in a form which is linear in $E(k)$ and $K(k)$. The results of carrying out this procedure for the leading coefficients are

$$I_1=(2^{\frac{1}{2}}\cdot 8GV^{\frac{1}{2}}/5\pi)\{2[(1-k^2)^2+k^2]E-(2-k^2)(1-k^2)K\}$$

$$I_2=(2^{\frac{1}{2}}\cdot 8GV^{\frac{1}{2}}/35\pi)[(1-k^2)(2+5k^2-8k^4)K-2(1-2k^2)(1+4k^2-4k^4)E] \quad (7b.3)$$

$$I_3=(2^{\frac{1}{2}}\cdot 8GV^{\frac{1}{2}}/315\pi)\times[2(128k^8-256k^6+135k^4-7k^2-1)E+(1-k^2)(128k^6-144k^4+15k^2+2)K].$$

Explicit expressions for higher order coefficients can be obtained, when required, by following the procedure outlined above.

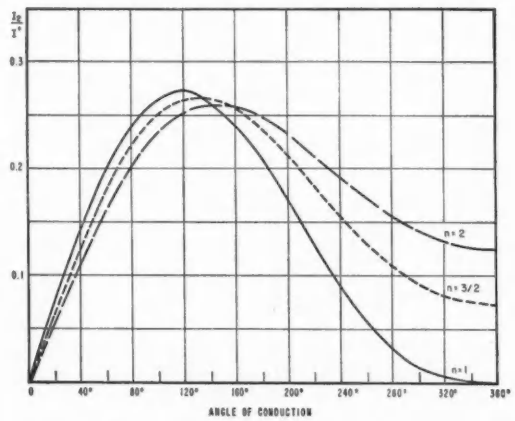

FIG. 4. The ratio of the second harmonic component of output current to the maximum value, as a function of the total angle of conduction, for $n=1$, $\frac{3}{2}$, and 2.

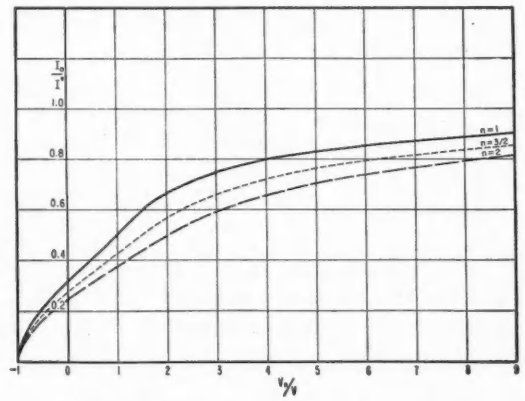

FIG. 6. The ratio of the dc component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for $n=1$, $\frac{3}{2}$, and 2.

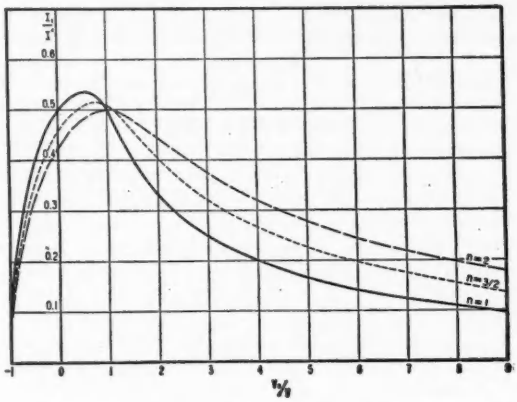

FIG. 7. The ratio of the fundamental component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for $n=1$, $\frac{3}{2}$, and 2.

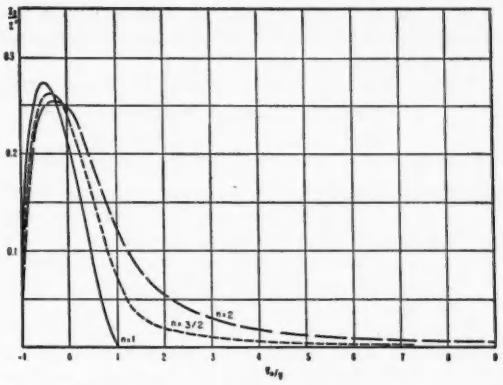

FIG. 8. The ratio of the second harmonic component of output current to the maximum value, as a function of the ratio of the steady component of the voltage to the peak value of the alternating component, for $n=1$, $\frac{3}{2}$, and 2.

GRAPHS

Results of the foregoing analysis are plotted in Figs. 1–8. Features of the discontinuous current case, for $n=1$, $\frac{3}{2}$, and 2, are shown in Figs. 1–4, the dependent variables for these figures being in order I/I', I_0/I', I_1/I', and I_2/I'. In each of these figures the independent variable is 2α, the total angle of conduction, results being shown for the entire possible range (0–360 degrees) of this variable.

In Figs. 5–8 the same dependent variables are plotted as functions of the variable $r=(V_0/V)$, the ratio of steady component of the applied voltage to peak value of the alternating component. In each of these figures the independent variable ranges from the value $r=-1$ to $r=9$. The portion of the range $r\geqslant 1$ corresponds to the case of continuous current, and the remaining portion, $-1\leqslant r\leqslant 1$, constitutes another version of the case of discontinuous current.

ACKNOWLEDGMENT

It is a pleasure to express my thanks to Mr. Walter Weber, formerly of this Laboratory, and to Mr. Henry D. Arnett for having carefully checked the calculations and for carrying out computations needed for the figures.

LIST OF SYMBOLS

i—instantaneous output current
v—instantaneous input voltage
G—constant of proportionality in the current-voltage law of the nonlinear device
n—exponent in the current-voltage law of the nonlinear device
V—peak value of alternating component of the voltage
V_0—value of the steady component of the voltage
$r=(V_0/V)$
θ—product of angular frequency and time
α—one-half of the total angle of conduction, when the current is discontinuous
I'—maximum value of the current
I—root-mean-square value of the current
I_0—value of the steady term in the Fourier series for the current
I_m—magnitude of coefficient of the mth term in the Fourier series for the current
$K(k)$—complete elliptic integral of the first kind
$E(k)$—complete elliptic integral of the second kind.

NOTES AND DISCUSSION

Further Note on Measurement of Wavelength with a Diffraction Grating

LOUIS R. WEBER AND DON L. HAMMOND
Colorado Agricultural and Mechanical College, Fort Collins, Colorado

IN 1937,[1] the senior author described an arrangement for locating the spectral lines of a mercury source as seen through a diffraction grating. Small lamps free to move along a two-meter scale enable measurements to be made with one-half percent error. An 866 rectifier tube was the source.

Units of this type have been used to date. Due to the low intensity of the source, a partially darkened room was necessary for viewing the spectrum and this condition prohibited the performing of other laboratory exercises at the same time that required more light.

With the development of the mercury RS sun lamp, a 275-w lamp of which operates directly from 110-volt lines with no external ballast, a spectrum can now be obtained with such brilliance that it can be seen in a fully illuminated laboratory.

Three units (Fig. 1) as now modified are being used in our general physics laboratory. The lamp is placed with the base down in an enclosure. The vertical rays strike two right angle prisms cut from window glass. The internally 90°—reflected rays from each prism pass through the opposite sides of the enclosure. The vertical leg of each prism becomes the equivalent of a brilliant vertical slit. A transmission grating is placed on each side of the unit at a distance of about two meters. Each piece of apparatus, therefore, accomodates two groups of students.

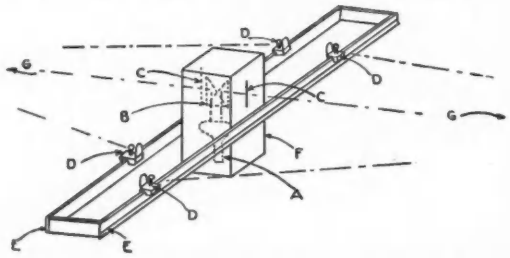

FIG. 1. *A*, RS sun lamp; *B*, window glass prisms; *C*, secondary slits; *D*, movable carriages carrying radio dial lamps; *E*, two-meter sticks; *F*, enclosure open at bottom and top for ventilation; *G*, transmission gratings two meters distance.

[1] L. R. Weber, Trans. Kans. Acad. Sci. **40**, 313 (1937).

On the Driving of Tent Stakes

JOHN S. RINEHART
Michelson Laboratory, U. S. Naval Ordnance Test Station, Inyokern, China Lake, California

THE very different behavior, when driven into hard ground, of a steel tent stake and of a wooden stake is an excellent example of the role that specific acoustic resistance can play in a practical situation. The problem is one that ought to be posed, though I have never known it to be, in connection with discussions of specific acoustic resistance in an intermediate acoustics class.

From experience we know that a steel stake can be driven into hard ground relatively easily and that after it has been driven, it will fit snugly. On the other hand, a wooden stake is hard to drive, the ground breaks up around it, and we end up with a loosely fitting stake. Each time one strikes the wooden stake one feels a strong ground shock. A similar shock is not felt when the steel stake is struck. The explanation for the different behaviors of the two stakes lies in a consideration of the relative specific acoustic resistances, i.e., density times velocity of sound, of steel, wood, and hard soil. The blow of the hammer sets up a transient stress wave in the stake. When this wave reaches the end of the stake that is in contact with the ground, part of the wave is reflected and part is transmitted into the ground. The relative amplitudes of the instantaneous stress transmitted, σ_T, and the stress reflected, σ_R, will be given approximately by[1]

$$\sigma_T/\sigma_R = 2\rho_2 c_2/\rho_1 c_1 - \rho_2 c_2,$$

where ρ_1 and c_1 are, respectively, the density and velocity of the wave in the stake and ρ_2 and c_2 are the density and the velocity of the wave in the ground.

It is apparent that if the product $\rho_2 c_2$ is small and much less than the product $\rho_1 c_1$, then most of the wave will be reflected at the junction and that the momentum imparted by the hammer will remain in the stake. The stake will acquire a high velocity and push itself into the dirt. If, on the other hand, the difference $(\rho_1 c_1 - \rho_2 c_2)$ is small, most of the wave will enter the ground and the stake will acquire little momentum. The transmitted wave tends to break up the soil. Substitution of reasonable values of ρ and c for steel, wood, and soil yield the following values of σ_T/σ_R: steel to soil, 0.1; and wood to soil, 29.

The above discussion simplifies to a considerable degree the complete mechanics of the progress of the stake. However, the role that specific acoustic resistance plays in its progess must be substantially that indicated.

[1] J. S. Rinehart, Am. J. Phys. **18**, 546 (1950).

Note on a Body Falling in a Resisting Medium of Variable Density

W. A. BOWERS
University of North Carolina, Chapel Hill, North Carolina

THE question has been asked recently in this journal,[1] whether the equation of motion for a body falling in a resisting medium of variable density (e.g., fall from large heights in the earth's atmosphere) can be integrated in closed form.

At least in the case of the second equation proposed by Bacon,

$$\ddot{y}=g-ke^{\beta y}(\dot{y})^2, \tag{1}$$

where y is the distance of fall (positive downward), g is acceleration of gravity, and $ke^{\beta y}$ is the coefficient of the square of the velocity in the resisting force per unit mass, (written thus to include the (approximate) exponential increase in density of the atmosphere with distance as the body falls), it is easy to find a first integral, in terms of tabulated functions, which gives the velocity as a function of distance. Although numerical integration (or various approximate methods) would be needed to find explicitly the time dependence of position and velocity, some interesting features appear from the first integral.

If we let $v=\dot{y}$, and regard y as the independent variable, Eq. (1) becomes

$$vdv/dy=g-ke^{\beta y}v^2, \tag{2}$$

which, on introducing the new independent variable $x=e^{\beta y}$, can be written,

$$dv^2/dx+(2k/\beta)v^2=2g/\beta x. \tag{3}$$

Since this is a linear first-order equation for v^2, it can be solved by standard methods to yield:

$$\begin{aligned} v^2&=(2g/\beta)e^{-\alpha x}\int_1^x e^{\alpha\xi}(d\xi/\xi),\\ &=(2g/\beta)e^{-\alpha x}\int_\alpha^{\alpha x} e^u(du/u), \end{aligned} \tag{4}$$

where we have used the symbol α in place of $2k/\beta$, and have imposed the initial condition: $v=0$ for $y=0$ or $x=1$. The integral appearing in the second line of Eq. (4) is the standard form of the exponential integral, for which numerous tables exist;[2] denoting, as usual, by $Ei(\alpha)$ the integral

$$\int_{-\infty}^{\alpha} e^u du/u,$$

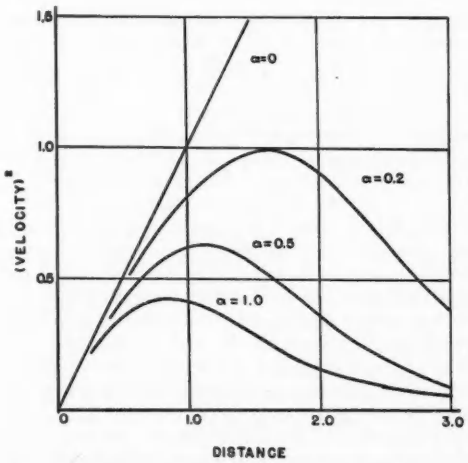

FIG. 1: (Velocity)^2 vs distance for values of α equal to 0, 0.2, 0.5, and 1.0. Ordinate: (velocity)^2 in units of $(2g/\beta)\cong 0.16$(km/sec)^2; abscissa: distance in units of $\beta^{-1}\cong 8$ km.

we may write Eq. (4), after substituting the original variable y, as follows:

$$v^2=(2g/\beta)\exp(-\alpha e^{\beta y})[Ei(\alpha e^{\beta y})-Ei(\alpha)]. \tag{5}$$

By taking $(2g/\beta)^{\frac{1}{2}}$ as the unit of velocity and β^{-1} as the unit of distance, this is reduced to a relation with a single parameter α.

We may note that if k (and hence α) vanishes, Eq. (4) gives $v^2=2gy$ as it should. For any nonvanishing value of α, however, the velocity, after initially increasing, will ultimately reach a maximum and begin to decrease again; this is, of course, reasonable physically in view of the exponentially increasing resistance term. In Fig. 1 we have plotted the square of the velocity against distance of fall for several values of α.

Finally we may consider what the order of magnitude of the various parameters is for situations of physical interest. First as regards β: experiment indicates that up to about 30,000 ft, the density does vary approximately exponentially with altitude, and that β has the value 0.11 km^{-1}. Above 30,000 ft, β increases to a value of approximately 0.15 km^{-1}. These data may be compared with the theoretical value of β for an isothermal atmosphere, which is Mg/RT, when M is the molecular weight of air, R the gas constant per mole and T the absolute temperature; if $T=273°$, this gives $\beta=0.125$ km^{-1}. Adopting this figure as a reasonable average, our unit of length, β^{-1}, has the value of 8 km, and our unit of velocity, $(2g/\beta)^{\frac{1}{2}}$, has the value 0.40 km/sec. Next as regards k: experimental values of resisting force are usually quoted in terms of the drag coefficient C_D, so defined that the resisting force is given by $(\frac{1}{2}\rho_m Sv^2)C_D$, where ρ_m is the density of the medium, S the cross-sectional area of the body, and v its velocity; then C_D is a dimensionless quantity. Comparing with Eq. (1), we see that $k=\frac{1}{2}\rho_h(SC_D/m)$, where m is the mass of the body, and ρ_h is the density of the air at the *initial* height h of the body. Thus we may write $k=\frac{1}{2}C_D(\rho_0/\rho)e^{-\beta h}(S/V)$, where ρ_0 is the air density at sea level, ρ the density of the body, and V is its volume. For spheres $S=\pi r^2$, $V=4/3\pi r^3$, and $C_D\leqq 0.4$ for values of Reynolds number from 10^3 to 10^5; using these values, together with $\rho_0=1.29\times10^{-3}$ g/cm^3, we have:

$$k\leqq 2\times10^{-4}\times(e^{-\beta h}/r\rho)\text{ cm}^{-1}.$$

Hence with these assumptions our parameter α has the value:

$$\alpha=2k/\beta=300e^{-\beta h}/r\rho,$$

where r is in cm and ρ in g/cm^3.

It depends strongly on the initial height as well as on the radius and density of the sphere; if a particular value of α corresponds to one case, the same value will apply to a case in which a body either smaller or less dense by a factor e is dropped from a height 8 km higher. We may conveniently write $\alpha=\alpha_0 e^{-\beta h}$ where $\alpha_0=300/r\rho$, so that α_0 depends only on the body; then if we introduce z, the height of the body above the ground, $(z=h-y)$, Eq. (5) becomes finally:

$$v^2=\exp(-\alpha_0 e^{-z})[Ei(\alpha_0 e^{-z})-Ei(\alpha_0 e^{-h})], \tag{6}$$

if we use the velocity and distance units mentioned above.

TABLE I. Maximum speed, v_m, height z_m above ground at which maximum speed is reached, and speed v_f on striking ground, for various spherical objects dropped from a height of 20 km.

Material	Object Radius in cm	v_m (km/sec)	z_m (km)	v_f (km/sec)
Steel Wood	15 150	0.40	7.0	0.33
Steel Wood	6 60	0.32	10.5	0.18
Steel Wood	3 30	0.26	13.3	0.12

In Table I we give numerical values for several quantities of interest for several cases of spherical objects dropped from a height of 20 km.

[1] R. H. Bacon, Am. J. Phys. 19, 64 (1951).
[2] E.g., Jahncke-Emde, *Funktionentafeln* (Dover Publications, New York, 1943).

Student Questionnaires as an Aid to Laboratory Teaching Techniques

WILLARD GEER
University of Southern California, Los Angeles, California

THE physics department of the University of Southern California uses a student questionnaire every semester in its laboratory classes in order to improve the quality of laboratory instruction. The improvement in instruction resulting from the use of this questionnaire justifies sharing with other schools a knowledge of its contents and of the results obtained from student response to it. Belief in its worth is shared by students, by the instructional staff, and especially by the laboratory director and the teaching assistants who conduct the laboratory sessions.

The questionnaire was used originally on account of a general dissatisfaction with the results of laboratory instruction and because the director of laboratories felt that supervision of lower division laboratories in the larger universities too often consisted of a few hurried visits by the class instructor and an occasional visit by the director of the laboratories in a "snoopervisorial" capacity. In such circumstances student reaction is heard only when individual students have grievances. The weakness of this practice lies in the fact that such procedure is unfair to the young laboratory instructor, in that the departmental staff hears about the quality of his work only from a few students carrying complaints, and may be unfair to the student because the instructor's undiscovered weaknesses can result in irreparable loss to some of his victims. Under the common practice of hit-and-miss visitation of the laboratory it is also possible to overlook unusual success on the part of certain laboratory assistants whose methods or solutions of teaching problems are of sufficient significance to share with all of the laboratory staff.

Trying to bear all these factors in mind, those responsible for the laboratory work at this institution have worked out the simple questionnaire printed herewith, revising it from time to time as additional facts seemed needed or as questions that proved irrelevant were discarded. The forms are generally given out about four to six weeks after the semester has started. The questions are handed to the students as they enter their large lecture classes and they return them filled out at the end of the hour. Data for each instructor are compiled by assigning to each item in the left-hand column a "plus one"; a "zero" for items checked in the center or satisfactory column; and a "negative one" for items checked in the column indicating an insufficiency. The instructor's average on each of the twelve items can then be compared with his own averages during any other semester or with averages of others on the laboratory staff. Individual consultations are arranged with the director of laboratories and the individual results are considered; each item of the questionnaire is discussed in detail with the aim of improving teaching performance, and to strengthen the items which show up as weaknesses. In case of weaknesses in item 3 (clarity of explanation) and in items 10, 11, and 12 (which measure closeness of contact with students), an especial effort is made to show the instructor by what means he may achieve better results. A low mark on item 3 is frequently subject to better interpretation if it is found that item 7 reveals the instructor to be talking "over the heads" of his students. When such a situation is encountered, remedial measures are advised, such as asking the instructor to direct questions to individual students at frequent intervals, inviting them to explain just what they wish to know or what they do not understand.

Valuable data for the laboratory director are found in the last three items. The best-liked experiments are generally difficult, but well-explained ones. The least valuable ones are either too easy or else too difficult. The answers to the "why?" question often revealed that the student learned nothing new. Many experiments were marked least valuable because of their poor relation to class work or because they appealed to the student as having no practical significance. Answers like these should be correlated with the results from item 4 and item 5 in order to interpret correctly the instructor's handling of his work. However, some experiments are in themselves weak and should be discarded and replaced by others more worthwhile. Not the least important value of this questionnaire lies in the help it gives in selecting experiments which are worthy of continuance in the laboratory program and others which are eminently unworthy and should be discarded.

From the student standpoint, the questionnaires are helpful in that students can say what they think without recrimination. Occasionally they become facetious, but underneath there runs a current of honesty and dignity that keeps the answers worthwhile. One student could not refrain from writing that the laboratory assistant "sits on the lab table as haughty as the Deity Himself and watches with scorn our clumsy efforts," which situation needs investigation both by the one responsible for the laboratories and by the young instructor himself. But by and large the student judgment shows itself ready to appreciate good teaching and to suggest ways which would improve the quality of the instruction. In the isolated cases of bitterness and sheer conflict of personalities, the questionnaire has

the salutary effect of allowing the student to "blow off steam" and the pressure is eased, making possible better adjustment in the laboratory experience.

When several hundred students must work under inexperienced and immature graduate students as laboratory instructors, many more clashes might be anticipated than are actually encountered. Without the questionnaire, students must bury their grievances or take them to some authority. In these circumstances the head of the department or the laboratory director feels poorly qualified to judge the laboratory instructor from the complaining student's story. With the questionnaire as evidence they can quickly learn whether the other students in the laboratory section agree with the complaining ones. Often it can be ascertained from this evidence that an incident is a personality clash and is subject to adjustment on that basis. But if the evidence indicates that the laboratory instruction needs improvement, experience shows that the laboratory instructors themselves are usually eager to effect the improvement.

1. Is the explanation of the experiment being done.....	too lengthy	satisfactory	too brief
2. Is the explanation of next week's experiment.....	too lengthy	satisfactory	too brief
3. Is the explanation itself.....	very clear	satisfactory	not clear
4. Does your instructor relate the experiment to your class work...	very well	satisfactorily	not well
5. Does your instructor show the practical significance of the experiment.....	very well	satisfactorily	not well
6. The stress on theory is.....	too great	satisfactory	too little
7. The level of explanation is.....	too high	satisfactory	too low
8. The instructor talks.....	too loud	satisfactorily	too low
9. You feel your instructor's standards are.....	too high	satisfactory	too lax
10. When you do not understand the theory of the experiment, he finds out.....	very quickly	soon enough	too slowly
11. When you are doing something wrong he finds out.....	quickly	soon	too late
12. You feel he knows the level of your work.....	very well	satisfactorily	not well

Your instructor's best features are:
Details he could improve are:
The two best experiments so far are:
The least valuable experiment was:
Why so?

AMERICAN JOURNAL OF PHYSICS VOLUME 19, NUMBER 9 DECEMBER, 1951

LETTERS TO THE EDITOR

Concerning Some Primitive Musical Instruments

ON a recent visit to the Morse Museum in Warren, New Hampshire, I found myself interested in the collection of primitive musical instruments which the owner, Mr. Ira Morse, has assempled from various parts of the so-called "primitive" world. By virtue of my interest Mr. Morse kindly consented to demonstrate several pieces, and this note is intended to report an observation on two in particular, a pair of African cymbals and one metal disk or plate which is excited by striking.

Regarding the cymbals, Mr. Morse explained that the players whom he encountered on his African safaris were able to produce the weirdest and most profound effects by certain arm and hand manipulation, and that he was at a loss to understand how a mere motion of the arms could effect the original sound so amazingly. This is what is done: a cymbal is taken in each hand, being held by a leather knob attached to its center. They are struck in the conventional way, face against face, with a substantial impact. Instantly the cymbals are separated and the arms are swung through graceful arcs and curves, up and down and around, the meanwhile the player moves up and down by bending at the knees. The original notes emitted by the first impact now suffer rapid changes in pitch by the advance and recession of the instruments upon the observer, and, in addition, a fantastic composite of interference effects is produced. The musical result is practically impossible to describe although it produces a most beautiful musical sensation. It would appear that physicists who are interested particularly in the physical basis of music would find here a rich vein for inquiry. Musicians might indeed experiment with the possibilities.

The case of the metal disk possesses other interesting aspects. The disk, of bronze, is about 6 inches in diameter and its shape may be likened to that of two bread-and-butter plates attached face-to-face, that is, thicker at the center than at the edges. This was originally drilled with two tiny holes about two inches apart and an inch in from the edge. It was then suspended on leather strings which went out from the holes at about an angle of 120° between them. Suspended thus, it was sounded by striking with a small mallet of leather, whereupon it emitted a series of very beautiful chords. Now when put on display, two additional holes were drilled in it, diametrically opposite the first two and symmetrically placed. It was thus held by four leather tongs. But now when struck it emitted only the deadest response; a blunt thud. Mr. Morse was grieved by this turn of events and asked me if the situation could be remedied, that is, if the piece could be restored. Obviously, the two new holes changed the mass distribution and the nodal circles and nodal radii. Could the original plate (or a close approximation to it) be restored by brazing or welding in these holes with the same material?

JULIUS SUMNER MILLER

Dillard University
New Orleans, Louisiana

Relative Causality?

IN his article entitled, "Causality, Relativity, and Language,"[1] Mr. Fein made the remark that the idea of causality was a relative one and depended upon the

frame of reference of the observer. He based this notion upon the fact that two events, not coincident in space and simultaneous in one frame of reference will, in another coordinate system moving with respect to the first, have a definite temporal order; and, in fact, depending upon the direction of the relative velocity of the reference frames, the two events can occur in either order in time. Therefore, he asserts, one observer can say that event A caused event B, and another observer can say just the opposite.

Mr. Fein evidently forgot what he said on the previous page, "We have learned, however, that the velocity of light is the limiting velocity." If one would keep this in mind he would see that if in one frame of reference events A and B were simultaneous, then there could never be a cause and effect relationship between them in *any* frame of reference. This is merely because their time separation (in any frame) is not enough to allow a light signal to travel the distance of their separation (in the same frame). For example, take his expression for $\Delta t'$

$$\Delta t' = \frac{v\Delta x/c^2}{[1-(v^2/c^2)]^{\frac{1}{2}}} = \frac{v\Delta x'}{c^2},$$

where

$$\Delta x' = \frac{\Delta x}{[1-(v^2/c^2)]^{\frac{1}{2}}}.$$

Then if L is the distance a light signal can travel during the time $\Delta t'$ in the O' frame, $L = v\Delta x'/c$. And since $v/c < 1$, one gets that $L < \Delta x'$. Hence one event cannot be the cause of the other in such a case.

LEE SPETNER

Applied Physics Laboratory
The Johns Hopkins University
Silver Spring, Maryland

[1] E. Fein, Am. J. Phys. 19, 211 (1951).

Causality, Relativity, and Language

MR. Spetner is quite right;[1] if two events are not separated by enough time to allow a light signal to connect them for one observer, this qualification will be consequent for all observers of the two events. That is, simply, that the space-time interval between the two events must be the same for all observers.

The point, however, misses the intent of the article. It was suggested that the concept of causality (indeed all the verbal language) is not derived from our experience with four dimensional space-time, but rather from a partitioning of space-time into three-dimensional-space-and-time. That, in particular, causality is not (nor can it be) defined by our psyche as existing only between events which may be connected by light signals.

The point may be clarified by an example to illustrate the subtle meaning of the term causality. Two balls, A and B, are separated by a distance of 4 seconds in terms of the fastest ball which can travel between them. We observe that A is set in motion and one second later B is set in motion. Can we connect these events causally if the fastest cause in this case would appear to take 4 seconds in travelling from A to B? Let us now imagine that between the balls is another pair of balls, a and b, triggered simultaneously by a spring. When this happens a strikes A in 2 seconds and b strikes B in three seconds, thereby showing how A moves one second before B. How is the "cause" transmitted? A implies a, a implies b, b implies B; therefore we may say A implies B, and the "cause" is transmitted in one second.

Will it be argued that A is not the cause of B because B's motion is not dependent on A's motion? Presume no knowledge of balls a and b. Now remove ball A. Ball a appears one second before ball B moves. Then a implies B. The events have changed and so has the "cause." It is evident that we cannot employ as a criterion for causality one which prescribes a change in position for either event. That would imply a new world configuration, and in that case we would have to again determine the spatial and temporal relationship of the events.

The cause-effect syndrome is but a means of ordering our perceptions. We also order our perceptions according to our "space-and-time sense." It can be argued that our "space-and-time sense" is trained by our recognition of certain relations between events which we call "causal relations." It can also be asserted that it is our "space-and-time sense" which leads us to postulate "causal relationships." It is a matter of prejudice to claim primacy for a "space-and-time sense," or dependency of "space-and-time sense" upon an ability to recognize "causal relations." Both "space-and-time sense" and "causal perception" are interdependent, each being only different aspects of the same property of our awareness. It is this property which suffers differently for different observers, and it is the expression of this property in terms of space and time and/or causality which is dependent upon the observer's particular partitioning of space-time.

ELIHU FEIN

Fisk University
Nashville, Tennessee

[1] Lee Spetner, Am. J. Phys. 19, 565 (1951) (preceding letter).

Again, we cannot infer that because sciences of things Divine and human are full of controversies and quarrels, therefore their whole subject matter is uncertain.—SPINOZA.

ANNOUNCEMENTS AND NEWS

Book Reviews

Engineering Thermodynamics. HERMAN J. STOEVER. Pp. 458, Figs. 140, 6×9¼ in. John Wiley and Sons, Inc., New York, 1951. Price $5.75.

This reviewer has for some years felt the necessity for a textbook in thermodynamics that better illustrated, portrayed, and otherwise presented the principles in such a way as to present a challenge to the average student of mechanical engineering. This book does not meet these specific requirements but does follow closely the middle-of-the-road character, which will find favor with a great number of teachers in thermodynamics.

The book is primarily intended for undergraduate students and is designed for a course covering a full year of study.

The author has divided the material into three basic parts: Part I is concerned with presenting the first law of thermodynamics, Part II with presenting the second law of thermodynamics, and Part III with presenting the basic applications of Part I and II.

The idea of such a method of presentation concurs favorably with the desires of many teachers in the field today. However, the presentation of gases, liquids, and vapors as one subject matter finds many opponents. It is still debatable whether the student is led into confusion when gases and vapors are to be treated in one assignment.

Many of the illustrations and examples are excellent. Some of the derivations are weak in accordance with the general trend now existing in the presentation of thermodynamics. The author has been careful and considerate in presenting illustrated problems after each basic principle, following later with problems covering the combined principles treated in the chapter. It is with particular favor that the reviewer found, following each chapter, the new symbols involved in each chapter of the text.

It is doubtful if the author has presented a sufficient challenge to the student for the book to be used for a full year's study. It is also a question as to whether the author goes far enough into the subjects covered in Part III to justify their presentation as individual subject matter. The basic principles involved are handled in a very acceptable manner for students who are not expected to go farther into the subjects.

C. H. PESTERFIELD
Michigan State College

Ultrasonics. P. VIGOUREUX. Pp. 163, Figs. 73. John Wiley and Sons, Inc., New York, 1951. Price $4.00.

For the increasing number of workers in ultrasonics as well as for interested observers, this book fills a definite need. Until its advent, one had to depend on the excellent and complete, but untranslated, 1950 edition of Bergmann for the latest compilation on the subject. The book introduces the reader to the techniques and the simpler aspects of the theory of propagation of high frequency compressional waves in fluids. The author attempts, and generally with success, to outline the principles involved, rather than give detailed descriptions of apparatus or procedure. For the details one must go to the periodical literature, his references to which are well-chosen and abundant, although not as complete as the Bergmann list. After a brief introduction in which is stressed the need for most work to be done at high frequencies, there are taken up, in order, the various means of generation, the theory of propagation, methods of observation, and discussion of velocity and absorption in gases and liquids. In the last is included a good account of the role played by molecular structure and the associated relaxation time. In the main the appeal is to the reader who seeks understanding rather than applications of principles.

It is appropriate to suggest that authors of books on ultrasonics might well display more discrimination in their selection of data on velocity for inclusion in tables. A comparison of work done on the same liquid by several workers would suffice in many cases to indicate whose data were least trustworthy. Some pieces of data on velocity in liquids included by Vigoureux are incorrect by as much as seven percent, when, even as far back as 1929, the error resulting from frequency control and measurement of distance was justifiably given by the earliest investigators as about one part in a thousand. Part of the cause for error is no doubt a lack of precise temperature control, which is important, but more often a lack of care in purification, physicists being prone to "take a bottle off the shelf" without doing a fractional distillation before measurement of velocity. It would be well also, if investigators chose some standard temperature, say 25° or 30°, for reporting measurements.

The book is remarkably free of mistakes, a repeated one however being the misspelling of the name of the well-known writer in the field, W. Schaaffs. The type used is large and readable. The figures are clear and invariably appropriate. All in all, it is a fine little volume for obtaining a condensed view of the entire field of ultrasonics.

ROBERT LAGEMANN
Vanderbilt University

Experimental Spectroscopy. RALPH A. SAWYER. Second Edition. Pp. 358+x. Prentice-Hall, Inc., New York, 1951. Price $5.00.

This revised edition of the author's 1944 book by the same name is 9 percent larger than the first edition, as a result of a more detailed treatment of certain sections which before had been discussed only briefly. The sections on infrared spectrometers and detectors, on transmission and reflection in the vacuum ultraviolet, and on spectrochemical light sources and methods have been expanded; and new material on light sources, resolving power and slit-width effects, new grating mountings, infrared equipment, and spectrochemical analysis has been added. Also, the chapter bibliographies have been revised and enlarged, and more recent references to the periodical literature are given.

Most of the recent books on spectroscopy have been detailed treatments of some special field of spectroscopy,

usually with little, if any, reference to the experimental apparatus or techniques. This book, on the other hand, places its major emphasis on a basic treatment of those principles and techniques which are common to practically all uses of spectroscopic apparatus. Consequently, it should be of value to all spectroscopists, whatever their special field may be. It is practically the only modern book available of its type.

The entire spectral range from the ultraviolet to the infrared is covered. A brief history of spectroscopy is given, and light sources and both prism and grating spectrographs are discussed. Chapters are included on the photographic process and on the determination of wavelength and spectral intensity. The final chapter on spectrochemical analysis will be of special interest to workers in industry, or other fields, who wish to consider the use of spectroscopy as an analytical tool.

It is no doubt easier to criticize a book than it is to write a book which will not be criticized, and Sibelius has said that "there has never been set up a statue in honor of a critic." The reviewer will, therefore, refrain from a recital of weaknesses, which will always be present, in some measure, in every book. It is his opinion that the strong points so far outweigh any weaknesses that may exist as to make this book an indispensable part of the library of every spectroscopist.

FORREST F. CLEVELAND
Illinois Institute of Technology

Quantum Mechanics of Particles and Wave Fields. ARTHUR MARCH. Pp. 292+x. John Wiley and Sons, Inc., New York, 1951. Price $5.50.

In this book, the author attempts the very difficult task of explaining the theory of wave fields to one who has had no quantum mechanics. The first part of the book is devoted to standard subjects such as (1) wave mechanics of a single particle, (2) wave mechanics of stationary states, (3) wave mechanics in matrix form, and (4) perturbation theory. The latter part of the book concerns wave fields, nuclear forces, and the idea of a fundamental length.

In teaching the Dirac radiation theory, it is vital that the presentation be made with the utmost clarity. One should emphasize two points, namely, (1) that second quantization is really a shorthand way of stating that particles obey either Bose-Einstein or Fermi-Dirac statistics, and (2) that the matrix elements for interaction of a photon with matter can be obtained only by comparison with classical theory, when the number of photons is large. The reviewer feels that the treatment in this book is clearer than most previous ones, but that it is still considerably short of the necessary excellence.

Ordinarily, a student will need to have worked with quantum mechanics for about a year before he is able to appreciate properly the theory of wave fields. He must have a thorough grounding in matrices and tensors, and should be quite familar with the Dirac wave equation for the electron. Some of this material is given in Chapter 6 of the present book, but there is not nearly enough. The digression on tensor calculus is so fast that relativistic corrections might be in order.

On rare occasions, the author slides over the development without giving all the steps. On page 156 there occurs the phrase "It is easily seen." This nearly always produces a mental block in the student, with a resultant waste of much of his time. On page 177 it is stated that "Such a vector is found in . . .," and the student is not told how one would be led to make such a discovery.

Incidentally, the reviewer takes issue with the statement on page 148: "The fact that Kellner and Hylleraas obtained for the (ionization) potential a value that differs from the experimental one only by a few parts per thousand verifies the accuracy of the quantum-mechanical He model" The exact solution of the wave equation for the ground state of helium has not as yet been obtained, and it is not legitimate to compare the experimental value of the energy with an approximate theoretical value when one does not know how close the approximation is. The reviewer hopes to obtain the exact solution in the near future with the aid of an electronic computer.

A student of field theory wants to know (1) how one is led to make assumptions (such as in β-decay) about interactions occurring between electrons, neutrinos, and nucleons and (2) what the connection between spin and statistics is. The author omits these topics.

The spirit of field theory can be gained if one just studies one particle, say a charged vector meson, rather than several different kinds of particles. The treatment should show how to set up the energy-momentum tensor, the Hamiltonian, the linear and angular momenta, and the magnetic moment.

The book will be valuable for those who wish to have both elementary quantum mechanics and field theory under one cover. The reviewer does not consider it the best source for just the theory of wave fields. In any case, the student will have to own more than one book, for there is still no adequate and comprehensive textbook which includes field theory.

JAMES H. BARTLETT
University of Illinois

A foolish consistency is the hobgoblin of little minds, adored by little statesmen and philosophers and divines.—RALPH WALDO EMERSON.

Author Index to Volume 19

In this index are listed names of authors and titles of their articles and notes. Abstracts of papers read at meetings are designated by the letter (A) after the number of the page on which the abstract may be found. Papers read at meetings for which an abstract has not been published are not included; however, they are indexed in the Analytical Subject Index. The symbol (L) after an entry signifies a Letter to the Editor.

Abraham, George. Graduate level laboratory training for government research scientists—487(A)
Aden, A. L. Microwave reflection from water spheres—163
Ainslie, D. S. Demonstration experiments in electromagnetic induction—232
——Motor-driven vibrator units for the measurement of capacitance—486(A)
Albright, John G. Review of *Methods in climatology*—65
Anderson, William R. Meeting of the Chicago Section—249
Andrewes, U. and T. J. Dillon. Photometric teaching methods using photoelectric cells—514
Andrews, C. L. Introduction to polarization of electromagnetic waves—159
——Correction to the treatment of Fresnel diffraction—280
——Review of *Electromagnetic waves and radiating systems*—477

Bachman, C. H. and J. B. Maginnis. Heat-sensitive color changes in some inorganic chemicals—424
Bacon, Ralph Hoyt. Motion relative to the surface of the rotating earth—52; Erratum—385(L)
——Motion of a particle through a resisting medium of variable density—64(L)
——Review of *Response of physical systems*—133
Baez, Albert V. Demonstration of the relationships between ac and dc voltage—399(A)
Bainter, Monica E. Meeting of the Wisconsin Section—397
Ballard, Stanley S. Proposal of the International Commission of Optics for international standardization of sign conventions and symbols in geometrical optics—122
——Proposed international standards of sign conventions and symbols for geometrical optics—328(A)
——Preprofessional orientation course for physics majors—330(A)
Band, William. Origin of the lighter elements—251(A)
Barbour, Ian G. John Wesley Hornbeck, 1881–1951—324
Barker, E. F. Daniel Leslie Rich, 1879–1951—390
Barr, E. Scott. Demonstration of Bernoulli's principle—248(L)
——(see Sellers, R. E.)—444(A)
Bartlett, James H. Review of *Quantum mechanics of particles and wave fields*—568
Bartunek, Paul F. Driver for the Calthrop resonance pendulum—57
Baudean, John E. (see Riess, Karlem)—116
Bayse, C. A. (see Ryan, J. L.)—400(A)
Bender, David F. Meetings of the Southern California Section—195; 398
Bendler, Harry M. Review of *The heavens above*—429
Bennett, G. S. Review of *Fundamentals of acoustics*—254
Betts, John G. Water runs up hill, or does it?—195(A)
Bikerman, J. J. Physical values which depend on the standard deviation of another value—58
Birnbaum, G. Microwave analog of the optical quarter-wave and half-wave plates—442(A)
Blackwood, Oswald. Review of *Reflections of a physicist*—66
——Apparatus wanted and for sale—384(L)
Blaha, F. Demonstration of emission and absorption of sodium vapor—130(L)
Blankenagel, John C. Lectures of the late Professor Benjamin W. Snow—60
Bock, Robert O. Meeting of the Colorado-Wyoming Section—483
Boggs, Isabel (see Stipe, Jr., J. Gordon)—443(A)
Bollman, Vernon L. Demonstration of a rocket-propelled airplane—195(A)
——New approach to the teaching of physics to premedical students—399(A)
Borg, S. F. Application of dimensional analysis—69
Bouyoucos, George J. Review of *The climate near the ground*—192
Bowers, W. A. Note on a body falling in a resisting medium of variable density—562
Bowersox, Ralph B. Review of *Atomic physics*—134
Brady, James J. Survey of photoelectric theories—397(A)
Brewer, Lyle. Seeing light and color—328(A)
Brewington, G. P. Comments on several friction phenomena—357
——(see Judkins, Roy L.)—380
Brown, Sanborn C. Review of *Applied nuclear physics*—429
Brown, T. B. Manual of advanced undergraduate laboratory experiments—145
——Laboratory manual to be produced as a memorial to Lloyd William Taylor—323(L)
——B-H curves with a lecture table magnetometer—442(A)
Brown, Jr., William Fuller. Electric and magnetic forces: a direct calculation. I—290; II—333
Buchta, J. W. John Wesley Hornbeck, recipient of the 1950 Oersted Medal for notable contributions to the teaching of physics—411
Bunce, Elizabeth T. (see Mohler, Nora M.)—170

Callihan, Dixon. Meeting of the Southeastern Section, American Physical Society—443
Carr, E. F. and C. Kikuchi. Physical model to demonstrate nuclear and paramagnetic resonance—486(A)
Carter, Howard A. Selective heating effect in the radiometer—386(L)
Caspari, Ernst. Lectures of Professor Robert Pohl in Göttingen—61
Caswell, R. S. Quantitative measurements of beta-radiation—400(A)
Chapin, E. K. Some analogous problems involving variable mass, moment of inertia, inductance, and capacitance—6
——Problems involving variable mass—436(L)
Chew, G. F. (See Moyer, B. J.)—17
——**and Burton J. Moyer.** High energy nucleon-nucleon scattering experiments at Berkeley—203
Christensen, F. E. Demonstration mass spectrometer—59
Clancy, Edward P. Method for changing the response of a system—190(L)
Cleveland, Forrest F. Review of *Experimental spectroscopy*—567
Clothier, Robert F. Problems—321(L)
Colwell, Robert C. Synthesis of atoms and their isotopes—481(A)
Connell, Jr., L. F. and H. C. Martin, Jr. Simple camera for taking x-ray powder patterns at elevated or reduced temperatures—127
Cook, C. Sharp. Capture of orbital electrons by nuclei (*K*-capture)—37
Cope, D. F. (see Hatch, A. J.)—137
Corben, H. C. Review of *Relativity physics*—132
——Review of *The special theory of relativity*—132
Corrsin, Stanley. Simple geometrical proof of Buckingham's π-theorem—180
Crawford, F. H. Use of curve differentials in thermodynamics—284
Cullity, B. D. One- and two-dimensional x-ray diffraction—500

Dalton, Charles C. Tracing of skew rays by analytical methods—250(A)
Danos, Michael. Has pressure direction?—248(L)
Decker, Fred W. Meetings of the Oregon Section—251; 397; 483
——Note on an inexpensive strong U-V source—251(A)
Dempster, Richard R. Does pressure have direction?—64(L)
——Status of the Oregon State College cyclotron project—397(A)
Dexter, Lewis A. and Robert A. Thornton. Analysis of transfer of training—538
Dhavale, D. G. Simple relation for an achromatic telescope objective—379

Dickinson, B. H. Louis Bevier Spinney, 1869–1951—325
——Meeting of the Michigan Teachers of College Physics—400
Dillon, T. J. (see Andrewes, U.)—514
Dixon, W. R. Electromagnetic moment of inertia—536
Dodd, Laurence Ellsworth. Old-time classroom recitation: can it be restored?—14
——Energy independent of mass in simple harmonic motion—189(L)
——Demonstration of the hydrostatic paradox: phase II—195(A)
——Closed differential pulley—399(A)
Dunning, Gordon M. Demonstrations with simple equipment—482(A)
Dutton, Frederic B. Review of *Chemistry visualized and applied*—393
Dwight, C. Harrison. Specialized physics—97

Eaton, V. Stop clock with magnetic fluid clutch—330(A)
Edwards, R. L. Review of *First principles of atomic physics*—67
Efron, Alexander. Spiral program in high school physics—331(A)
Eisner, Leonard. Sinple method for locating principal points—474
Eldridge, John A. Energy density in a gravitational field—63(L)
Eller, Willard H. Short radius optical lever for use with Young's modulus apparatus—379
Erickson, Willard L. Moment of inertia experiment—401
Everhart, Edgar. Experiment for the direct measurement of magnetostatic fields—474

Farwell H. W. Forms of Cartesian ovals in an optical range—454
Feeny, H. (see Whitmore, F. C.)—442(A)
Fein, Elihu. Causality, relativity, and language—211; 439(L); 566(L)
Fisher, L. H. Representation of the static polarization of rigid dielectrics by equivalent charge distributions—73
Forster, George. Education?—Or merely training?! VIII, A demonstration that resolved a dilemma—195(A); IX, Potential difference *vs* potential gradient—398(A)
Fowler, Charles A. Simple polarized light demonstration—398(A)
Fox, Edwin S. Improved apparatus for demonstrating an oscillatory discharge—486(A)
Fritchle, Frank P. Fitch's apparatus for the measurement of the thermal conductivity—475(L)
Fussler, Karl H. (see Straley, Joseph W.)—313

Gaines, Newton. Experimental derivation of the Helmholtz dissonance curve using modern electrical apparatus—487(A)
Gale, Grant O. Constructive and destructive interference—321
Geer, Willard. Student questionnaires as an aid to laboratory teaching techniques—564
Gerhard, S. L. Slugging out a case for the pounders—a reply—440(L)
Gilvarry, John J. Maxwell's thermodynamic relations—131(L)
Goldowski, N. Introduction of physics to freshmen—444(A)
Goldsmith, Norris W. Development of a course "Backgrounds of Science"—330(A)
Goldstein, Herbert. Classical motion of a rigid charged body in a magnetic field—100
Green, L. C. Putting the physics in the teaching of astrophysics—442(A)
Groetzinger, Gerhart. Review of *Photons and electrons*—192

Haden, Harley J. Which is the more accurate?—189(L)
Hahn, Jr., T. M. University of Kentucky Van de Graaff generator—250(A)
——(see Rayburn, Charles C.)—400(A)
Ham, L. B. Review of *Adventure into the unknown*—430
Hammond, Don L. (see Weber, Louis R.)—562
Hanau, Richard. Meetings of the Kentucky Section—250; 400
——Propagation of errors in the elementary course—382(L)
Harbour, Dean H. Alternating current experiments at fifty cents each—399(A)
Harvalik, Zaboj V. Modified Cotton balance—128
Hatch, A. J. and D. F. Cope. Flashback teaching technique applied to a block-and-gap physics course—137
Hause, C. D. Proceedings of the American Association of Physics Teachers; East Lansing meeting—484
——(see Smith, A. E.)—488(A)
Haynes, Sherwood K. Review of *Microwave electronics*—133
Hazen, Wayne E. Review of *The principles of cloud-chamber technique*—427
Hazzard, George W. Dispersion and resolving power of prism spectrometers—235
——Suggestions for the improvement of physics teaching in colleges and universities—374
Hendricks, J. C. Meeting of the Indiana Section—399
Herman, J. C., B. V. Rhodes, and M. S. McCay. Design and performance of classroom and laboratory wind tunnels—443(A)
Hilton, Wallace A. Mercury light source—248(L)
Hitchcock, Richard C. Demonstrating harmonics and beats—329(A); 445
Holm, Dale Marvin. Neutron detection methods and the demonstration of a BF_3 counter—397(A)
Hopson, J. E. Investigation of gas amplification in a proportional counter—250(A)
Hornbeck, John W. Some reflections on the teaching of physics—412
Hough, James. Fundamental problems of experimental physics—489
Howey, Joseph H. Analysis of an *R-C* oscillator—85
Hussain, Zahur. Newton's law of attractions—146
——About an argument of Newton—197

Jacobs, Donald H. Review of *Technical optics, volume II*—326
Jehle, Herbert. Review of *Albert Einstein—philosopher-scientist*—252
Jenne, Everett K. Demonstration of the diffraction and interference of sound—397(A)
Jeppesen, Myron A. Experiments with doubly refracting crystals—81
Judkins, Roy L. and G. P. Brewington. Vapor pressure-temperature apparatus—380

Katz, Robert. Radioactive disintegration—389(L)
Kauffman, G. E. C. Physics in the civil defense program—442(A)
Kelley, James B. Compressible flows—237
Kelly, W. C. Meeting of the Western Pennsylvania Section—481
——Seminar for physics majors—482(A)
Kennon, W. L. Apparatus for the study of centrifugal force and rotational inertia for use in general physics laboratory—443(A)
Kern, B. D. (see Ryan, J. L.)—400(A)
Kikuchi, C. (see Carr, E. F.)—486(A)
Kinsey, W. H. and R. A. Rhodes, II. Laboratory examination for general college physics—246
Klebba, Arthur A. and Henry Stommel. Simple demonstration of Coriolis force—247
Klein, Martin J. Order parameters—153
Klinkenberg, A. and G. J. Sleutelberg. Extended Bernoulli equation—435(L)
Klopsteg, Paul E. Annual report of the treasurer—331
Knauss, Harold P. and Paul R. Zilsel Magnetically maintained pendulum—318
Koehler, W. F. Laboratory experiment on the determination of γ for gases by self-sustained oscillations—113
——Review of *Heat and temperature measurement*—194
Kofsky, Irving L. Yo-yo technics in teaching kinematics—126
——Atwood's machine and the teaching of Newton's second law—354
Korff, Serge A. Cosmic-ray neutrons—226
Kretschmar, George G. Improved mechanical equivalent of heat experiment—509
Kruglak, Haym. Behavior objectives for laboratory instruction—223
——(see Wall, C. N.)—546

Lagemann, Robert. Review of *Ultrasonics*—567
Lange, K. O. (see Watson, F. D.)—250(A)
Laufer, Arthur R. Orientation of paramagnetic and diamagnetic rods in magnetic fields—275
Lax, Melvin. Review of *Fundamentals of quantum mechanics*—478
Lee, J. C. (see Schlegel, R.)—470
Lefler, R. W. Meeting of the Indiana Section—196
Liller, P. R. (see Whitmore, F. C.)—442(A)
Lindsay, George. Pressure energy and Bernoulli's principle—487(A)
Lindsay, R. B. Review of *The nature of physical reality*—67
Little, Noel C. Unified approach to physics—351
Lorentz, Lilly (see Mohler, Nora M.)—170
Loring, Ralph A. Student spectograph from surplus equipment—329(A)
——Lenses for laboratory-built spectrographs—487(A)
Luchak, George. Fall of a particle through the atmosphere—426

Maginnis, J. B. (see Bachman, C. H.)—424
Maney, Charles A. Study of sliding friction—487(A)
Margenau, Henry. Review of *Introduction to the study of physics*—433

Martin, Jr., H. C. (see Connell, Jr., L. F.)—127
Mason, John B. (see Tindal, Charles H.)—382
McCarthy, John J. World trends in the publication of physical research, 1938–1948—79
McCay, M. S. (see Herman, J. C.)—443(A)
McKeehan, L. W. Teaching by publication—9
McKown, Daniel C. (see Owen, George E.)—188
McMillen, J. H. Meeting of the Chesapeake Section—441
Meijer, R. R. Interesting type of problem in physics—442(A)
Menger, Karl. Mathematics of elementary thermodynamics—476(L)
Merritt, Thomas P. Physics via television—386(L)
——Review of *The new physics*—434
Metzgar, K. J. Acoustic interferometer—482(A)
Michener, W. H. Review of *Physics, its laws, ideas, and methods*—392
Miller, Jr., Franklin. Demonstration of phase shift using voltmeters—366
Miller, Julius Sumner. Weight of a falling chain—63(L)
——Teaching device—130(L)
——Departure in general physics laboratory procedure—190(L)
——What constitutes a laboratory examination?—191(L)
——Lissajous figures in Melde's experiment—248(L)
——Electric discharge in air at reduced pressure—330(A)
——Extension of the falling chain problem—383(L)
——Dropping a stone down a shaft—436(L)
——Concerning classroom recitation—476(L)
——Concerning some primitive musical instruments—565(L)
Mohler, Nora M., Lilly Lorentz, and Elizabeth T. Bunce. Elementary laboratory for premedical students—170
Montgomery, D. J. On stirring a cup of tea—477(L)
Mooney, Frank. Testing the Rayleigh resolving power criterion—130(L)
——Snell's law equivalent to the conservation of tangential momentum—385(L)
Moore, W. Cullen. Mechanical model for demonstrating Fermat's principle—1
——Wave fronts, attenuation, and diffraction—331(A)
Morris, Lloyd W. Lecture demonstration of coupled systems employing selsyn motors to provide variable coupling—443(A)
——(see Yeadon, Thomas J.)—444(A)
Moyer, B. J. and G. F. Chew. Collisions of high energy nuclear particles with nuclei—17
——(see Chew, Geoffrey F.)—203
Mrozowski, S. Review of *Molecular spectra and molecular structure*—390
Munick, Raymond J. Causality, relativity, and language—438(L)
Murphy, Richard D. Practical laboratory experience—384(L)
Myers, Jr., Ora E. Studies of transmission zone plates—359
Myers, Vernon. Effect of a constant force on a particle in a box—329(A)

Naymik, Daniel A. Knotty masspoint *versus* the rigid tinplate—519
Nier, Alfred O. and R. B. Thorness. Automatic chart plotter for lecture room demonstrations—416
Noll, Victor H. Review of *Methods and materials for teaching general and physical science*—478
Noll, Waldemar. Demonstration experiments—329(A)
Nye, H. A. Review of *Introduction to electricity and optics*—194

Osgood, Thomas H. Review of *Vacuum equipment and techniques*—431
Owen, George E. Criteria for choosing laboratory experiments—381
——**and Daniel C. McKown.** Experiment illustrating the elliptic integral of the first kind—188

Page, Thornton. Starting point in physics teaching—388(L)
——Review of *The origin of the earth*—432
Pake, G. E. Review of *Introductory nuclear physics*—252
Palmer, Frederic. Review of *The friction and lubrication of solids*—428
Paton, R. F. Vote on the new constitution and by-laws—312
——Report of the secretary—331
Patterson, Louise Diehl. Thermometers of the Royal Society—523
Payne, W. T. Theorem on deviation in thin lenses—57
Peach, Harry. Lumen—322(L)
Peck, Jr., R. A. and Paul Stelson. Laboratory exercise in nuclear emulsion technique—48
Pesterfield, C. H. Review of *Engineering thermodynamics*—567
Petry, Robert. Study of subjects for motion pictures for teaching physics—487(A)
Phelps, Everett R. Science on television—387(L)
Phelps, John. Mathematical emphasis in undergraduate physics—122
Pierce, W. M. Chladni plate figures—436(L)
Piesset, Milton S. Review of *Hydrodynamics*—479
Pockman, Leonard T. Newtonian mechanics and the equivalence of gravitational and inertial mass—305
Potter, James G. Study tips for the physics student—488(A)
Purdy, Mabel A. Meeting of the Physics Club of Philadelphia—251

Railsback, O. L. Meeting of the Illinois Section—196
Rayburn, Charles C. and T. M. Hahn, Jr. Fast coincidence analyzer—400(A)
Raymond, Richard C. Well-informed heat engine—109
——Review of *Thermodynamics*—193
Reese, Herbert M. Freezing in water pipes—425
Rhodes, B. V. (see Herman, J. C.)—443(A)
Rhodes, II, R. A (see Kinsey, W. H.)—246
Riess, Karlem and John E. Baudean. Experiments in viscous fluid flow—116
Riggs, Cecil O. Demonstrations of nuclear ray tracks—482(A)
Rinehart, John S. On the driving of tent stakes—562
Robertson, J. K. Teaching of physics to premedical students—131(L)
Rodgers, Eric. University of Alabama Physics Building Dedication Meeting—255
Rogers, Max T. Review of *Advances in radiochemistry*—480
Rubendall, Edward I. Anomaly in the determination of the cofficient of kinetic friction—482(A)
Ryan, J. L., B. D. Kern, and C. A. Bayse. Focusing problems and voltage stabilization of the Kentucky Van de Graaff generator—400(A)
Ryerson, Joseph L. Solution of differential equations by electrical analog computers—90

Salzberg, Bernard. Output current of a nonlinear device—555
Sartain, Carl C. Designation of a thyratron tube—389(L)
——Laboratory course in modern physics—443(A)
Satterly, John. Use of Fitch's apparatus for the measurement of the thermal conductivity of thin slabs of poorly conducting materials—132(L)
——Replies to inquiring letters—191(L)
——Simple dynamics and the value of g—322(L)
——Falling chains—383(L)
——Review of *Physics*—393
——C. V. Boys' rainbow cup and experiments with thin films—448
——Rocking of an elliptic band—511
Schepler, H. C. and A. N. Smith. Basic principle for the telescope and microscope—129
Schlegel, R. and J. C. Lee. Are x-ray tube demonstrations safe?—470
Schmookler, Jacob. Review of *Economic aspects of atomic power*—480
Schoepfle, G. K. Review of *Electricity and magnetism*—193
Schultz, H. L. and W. G. Wadey. Laboratory course in electronics—214
Sears, Francis W. Three lecture demonstrations—329(A)
Seeger, R. J. On aerophysics research—459
Seifert, Howard S. Projection of small scale phenomena—195(A)
Sellers, R. E. and E. Scott Barr. Laboratory experiment on radial heat flow—444(A)
Shaw, Robert S. Who writes for the science teachers' magazines?—322(L); 331(A)
Shearin, Paul E. (see Straley, Joseph W.)—313
Shirley, John W. Harvard case histories in experimental science; the evolution of an idea—419
——Binary numeration before Leibniz—452
——Early experimental determination of Snell's law—507
Simon, A. W. Application of the generalized area-moment propositions—35
Skolil, Lester L. Use of the coulomb in electrostatic problems—245
Slack, Francis G. Fellow members please read the fine print—389(L)
Slater, J. C. Electron theory of solids—368
Sleator, W. W. Potential against resistance—a graphical review—262
——Center of percussion—486(A)
——Longitudinal waves—487(A)
Slepian, Joseph. Lines of force in electric and magnetic fields—87
Sleutelberg, G. J. (see Klinkenberg, A.)—435(L)
Smith, A. E. and C. D. Hause. Fraunhofer multiple-slit diffraction patterns with infinite sources—488(A)

Smith, A. N. (see Schepler, H. C.)—129
Smith, Lloyd P. Engineering physics at Cornell—174
Smith, Orrin H. Inexpensive high resistance voltmeter—244
Spence, B. J. Review of *The autobiography of Robert A. Millikan*—65
Spence, R. D. Review of *Wave guides*—326
Spetner, Lee. Relative causality?—565(L)
Squire, William. Motion of a particle through a resisting medium of variable density—426
Steison, Paul. (see Peck, Jr., R. A.)—48
Stewart, G. W. Review of *From the life of a researcher*—427
Stipe, Jr., J. Gordon and Isabel Boggs. Third-semester physics course—443(A)
Stolov, Harold L. Semidiurnal tidal oscillation of the earth's atmosphere—329(A): 403
Stommel, Henry (see Klebba, Arthur A.)—247
Straley, Joseph W. Use of recording-controlling instruments in the intermediate heat laboratory—443(A)
——, **Karl H. Fussler, and Paul E. Shearin.** Elementary laboratory in modern physics—313
Sutton, Richard M. Selected simple demonstration experiments in mechanics—330(A)
Swann, W. F. G. Teaching of physics—182

Taylor, Julius H. Apparatus for measuring the acceleration due to gravity—245
Thomsen, John S. Problems involving variable mass—435(L)
——Mathematics of elementary thermodynamics—476(L)
Thorness, R. B. (see Nier, Alfred O.)—416
Thornton, Robert A. (see Dexter, Lewis A.)—538
Throw, Francis E. Matter of terminology; the kilocalorie and the kilomole—436(L)
Tindal, Charles H. and John B. Mason. Effect of aeration on the viscosity of water—382
Tobey, A. R. Chronology of modern physics—167
Trainor, L. E. H. (see Wall, C. N.)—546
Trent, Horace M. Review of *Laplace transformation*—391
——Implication of the Laplace transtormation—437(L)

Ufford, C. W. Teaching atomic physics to engineers and other insolvable teaching problems—442(A)

Van Name, Jr., F. W. Illustrating the regularity of alpha-radioactivity—230
Varner, William R. Efficiency in teaching measurements—397(A)
Voelker, C. H. Physics of the Chesapeake section—442(A)
——Training in undergraduate physics—442(A)

Wadey, W. G. (see Schultz, H. L.)—214
Walker, A. C. Review of *Crystal growth*—430
Wall, C. N., H. Kruglak, and L. E. H. Trainor. Laboratory performance tests at the University of Minnesota—546
Wang, Shou Chin. Graphical solution for the series impedance equivalent to two impedances in parallel—178
Wangsness, Roald K. Bonus questions and recommended reading—475(L)
Warburton, F. W. Functional definitions and the meaning of electricity—398(A)
Watson, F. D. and K. O. Lange. Sonic determination of air temperature—250(A)
Weber, Louis R. and Don L. Hammond. Further note on measurement of wavelength with a diffraction grating—562
Wender, Benjamin H. Some laboratory tests—438(L)
White, Marsh W. Enrollments and degrees awarded to physics majors—27
——Employment of physicists by the Department of the Army—257
Whitmore, F. C., P. R. Liller, and H. Feeny. Ten-channel time sequential analyzer—442(A)
Williamson, Charles. Sensitive arrangements of the Wheatstone bridge—123
——Projects in electricity laboratory—482(A); 486(A)
Winans, J. G. Slugging out a case for the pounders—439(L)
——Vector quantities in introductory physics—488(A)
Woodcock, Jr., Wilson W. Synthetic rutile—323(L)
Worth, Donald. Experiment demonstration to determine rifle bullet velocity—250(A)

Yeadon, Thomas J. and Lloyd W. Morris. Multiple frequency standard employing a modulated television-type raster for comparison of frequencies—444(A)

Zemansky, M. W. Regelation of ice—442(A)
Zilsel, Paul R. (see Knauss, Harold P.)—318

The different views as to how men are to be arranged into the good society . . . are merely the expression of individual preferences, which in turn are the results of different environmental and hereditary conditions. All *these theories, we suggest, are probably mistaken, because they are erected on foundations that consider only a small fraction of the factual data now available. If one happens to be right, it is nothing more than an inspired guess.*

In much the same way, for two hundred years before 1850, such learned journals as the Philosophical Transactions of the Royal Society *had their pages crammed with methods for attaining perpetual motion. Machines were being invented, not by crackpots, but by the best scientific minds of the generation—men of the caliber of Newton, Huygens, and Hooke. An immense amount of labor went into the design and construction of such machines, and into the individual criticism of theories and mechanical embodiments that accompanied their publication. Toward the middle of the nineteenth century, this debate (which had been carried on in a desultory fashion since the time of the ancient Greeks, and intensively since the rebirth of physics that came with the Renaissance) came to an abrupt end. Such work as Count Rumford's experiments to show the equivalence of heat and work, and Joule's numerical calculation of the mechanical equivalent of heat permitted a great generalization: the law of conservation of energy. This general law permitted men to solve questions dealing with energy on a theoretical level, without building a mechanical model, and without wasting time on the criticism of individual mechanical embodiments. The relatively advanced state of the physical, as opposed to the social, sciences is largely the result of this law. . . .*

Today we stand badly in need of some general laws dealing with the dynamics and statics of society—something analogous to the "law of conservation of energy" and "principle of least action" in physics. . . . If we had such laws we would be able for the first time to give direction *to research. We would know immediately what was possible and what was not possible, without having to perform costly experiments to establish the field.*—E. W. Leaver and J. J. Brown, Science **114**, 379 (1951).

Analytic Subject Index to Volume 19

The titles of articles are disregarded, the entries being based on analyses of the contents of the articles. The symbol (A) designates an abstract of a paper read at a meeting; (T) designates title only and (L) designates a Letter to the Editor.

To facilitate reference to any desired subject, the index is divided into sections arranged alphabetically. The titles of these sections are as follows:

Accelerators
Aerophysics
American Association of Physics Teachers
American Physical Society
Apparatus
Cosmic rays
Cosmography
Courses
Demonstrations
Department administration, maintenance, and activities
Education, physics and science
Electricity and magnetism
Employment of physicists
Experiments
General education
General physics, educational aspects
General physics, instructional techniques
Geophysics
Heat and thermodynamics
History and biography
Industrial and governmental research
Laboratory arts and techniques
Laboratory organization
Language
Light
Mathematics
Mechanics
Mesons
Microwaves
Modern physics
Nuclear physics
Philosophy of science
Properties of matter
Radio and television
Reports, announcements, and news
Rockets
Secondary-school physics
Social and economic aspects of science
Sound
Teacher training
Testing, theory and techniques
Textbooks
Units, dimensions, and terminology
Visual materials and methods
X-rays

Accelerators

Focusing problems and voltage stabilization of the Kentucky Van de Graaff generator, J. L. Ryan, B. D. Kern, and C. A. Bayse—400(A)

Operation of the University of Kentucky 1-Mev Van de Graaff generator, Homer H. Givin, T. M. Hahn, Jr., George C. Patterson, and L. W. Cochran—400(T)

Status of the Oregon State College cyclotron project, Richard E. Dempster—397(A)

University of Kentucky Van de Graaff generator, T. M. Hahn, Jr.—250(A)

Van de Graaff generator for demonstration purposes, Richard H. Waters—195(T)

Aerophysics

Design and performance of classroom and laboratory wind tunnels, J. C. Herman, B. V. Rhodes, and M. S. McCay—443(A)

Earth's atmosphere, semidiurnal tidal oscillation of, Harold L. Stolov—329(A); 403

Effect of aeration on the viscosity of water, Charles H. Tindal and John B. Mason—382

Fall of a particle through the atmosphere, Goerge Luchak—426

On aerophysics research, R. J. Seeger— 459

Sonic determination of air temperature, F. D. Watson and K. O. Lange—250(A)

American Association of Physics Teachers

Fellow members please read the fine print, Francis G. Slack—389(L)

Lloyd William Taylor, laboratory manual as a memorial to, T. B. Brown—323(L)

Meeting at East Lansing, June 26–28, 1951: program and abstracts, C. D. Hause—484

Meeting at New York, February 1–2–3, 1951: program and abstracts—328; treasurer's report, Paul E. Klopsteg—331; secretary's report, R. F. Paton—331

Necrology: John Wesley Hornbeck, 1881–1951—324; Louis Bevier Spinney, 1869–1951—325; Daniel Leslie Rich, 1879–1951—390

New members of the association—47; 121; 255; 327; 395; 434; 481; 545

Presentation of the Oersted Medal of the American Association of Physics Teachers to Professor John W. Hornbeck, Duane Roller and J. W. Buchta—328(T), 411; Some reflections on the teaching of physics, John W. Hornbeck—328(T), 412

Richtmyer Memorial Lecture—Electron theory of solids, John C. Slater—328(T); 368

Section news: Chesapeake, J. H. McMillen—441; Chicago, William R. Anderson—249; Colorado-Wyoming, Robert O. Bock—483; Illinois, O. L. Railsback—196; Indiana, R. W. Lefler—196; J. C. Hendricks—399; Kentucky, Richard Hanau—250, 400; Oregon, Fred W. Decker—251, 397, 483; Southern California, David F. Bender—195, 398; Western Pennsylvania, W. C. Kelly—481; Wisconsin, Monica E. Bainter—397

Summer meeting of the Association at Michigan State College—243

Vote on the new constitution and by-laws, R. F. Paton—312

American Physical Society

Address of the retiring president—Molecular beam researches in nuclear and electronic physics, I. I. Rabi—328(T)

Section news: Southeastern, Dixon Callihan—443

Apparatus

Acoustic interferometer, K. J. Metzgar—482(A)

Apparatus for measuring the acceleration due to gravity, Julius H. Taylor—245

Apparatus for the study of centrifugal force and rotational inertia for use in general physics laboratory, W. L. Kennon—443(A)

Apparatus wanted and for sale, O. Blackwood—384(L)

Atwood's machine and the teaching of Newton's second law, Irving L. Kofsky—354

Automatic chart plotter for lecture room demonstrations, Alfred O. Nier and R. B. Thorness—416

Balanced-beam, drift-eliminated recording microphotometer, A. H. Budlong—398(T)

BF_3 counter, demonstration of, Dale Marvin Holm—397(A)

Classroom and laboratory wind tunnels, design and performance of, J. C. Herman, B. V. Rhodes, and M. S. McCay—443(A)

Closed differential pulley, Laurence E. Dodd—399(A)

C. V. Boys' rainbow cup and experiment with thin films, John Satterly—448

Demonstration mass spectrometer, F. E. Christensen—59

Driver for the Calthrop resonance pendulum, Paul F. Bartunek—57

Dynamic hysteresis loop tracer, T. A. Benham—136(T)

Electrical analog computers, solution of differential equations by, Joseph L. Ryerson—90

Electronic diffraction analyzer, K. R. Symon—400(T)

Elliptic mirror for lecture demonstration, J. Smithson and W. T. Fenhagen—442(T)

Fast coincidence analyzer, Charles C. Rayburn and T. M. Hahn, Jr.—400(T)

Fitch's apparatus for the measurement of thermal conductivity, John Satterly—132(L); Frank P. Fritchle—475(L)

Improved apparatus for demonstrating an oscillatory discharge, Edwin S. Fox—486(A)

Improved electron projection microscope, Frank Grundhauser—251(T)

Inexpensive high resistance voltmeter, Orrin H. Smith—244

Inexpensive strong U-V source, Fred W. Decker—251(A)

Ionization chamber, calibration of, Duis D. Bolinger—397(T)

Laboratory-built spectrographs, lenses for, Ralph A. Loring—487(A)
Large-sized apparatus in lecture demonstrations in physics, W. H. Kadesch—483(T)
Lauritsen quartz fiber electroscope, student experiments with, G. Karioris—398(T)
Mechanical model for demonstrating Fermat's principle, W. Cullen Moore—1
Mercury light source, Wallace A. Hilton—248(L)
Modern instrumentation needed at the undergraduate level, M. E. Hufford—399(T)
Modified Cotton balance, Zaboj V. Harvalik—128
Modified Wilson cloud chamber, George C. Patterson—251(T)
Motor-driven vibrator units for the measurement of capacitance, D. S. Ainslie—486(A)
Multiple frequency standard employing a modulated television-type raster for comparison of frequencies, Thomas J. Yeadon and Lloyd W. Morris—444(A)
Osmosis, apparatus for demonstrating, H. D. Smith—400(T)
Particle counters, Robert Walker—399(T)
Pendulum, magnetically maintained, Harold P. Knauss and Paul R. Zilsel—318
Photoelectric cells, photometric teaching methods using, U. Andrewes and T. J. Dillon—514
Physical model to demonstrate nuclear and paramagnetic resonance, E. F. Carr and C. Kikuchi—486(A)
Proportional counter, investigation of gas amplification in, J. E. Hopson—250(A)
Radiometer, selective heating effect in, Howard A. Carter—386(L)
Recording-controlling instruments, use in heat laboratory, Joseph W. Straley—443(A)
Rocket-propelled airplane, demonstration of, Vernon L. Bollman—195(A)
Scintillation counter, detection of soft x-rays with, Lyman A. Webb, Ronald S. Paul, and Francis E. Dart—483(T)
Shock tube and its functions, Walter Bleakney and Wayland C. Griffith—486(T)
Short radius optical lever for use with Young's modulus apparatus, Willard H. Eller—379
Simple camera for taking x-ray powder patterns at elevated or reduced temperatures, L. F. Connell, Jr. and H. C. Martin, Jr.—127
Solid boron neutron detector, design and construction of, Dale M. Holm—483(T)
Some uses of surplus equipment, A. D. Hummell—196(T)
Stop clock with magnetic fluid clutch, V. Eaton—330(A)
Stroude and Oates induction bridge, W. H. Hyslop—483(T)
Student spectrograph from surplus equipment, Ralph A. Loring—329(A)
Telescope and microscope, basic principle for, H. C. Schepler and A. N. Smith—129
Ten channel time sequential analyzer, F. C. Whitmore, P. R. Liller, and H. Feeny—442(A)
Thermometers of the Royal Society, Louise Diehl Patterson—523
Thin-windowed Geiger-Müller counters, design and characteristics of, R. L. Purbrick, Lawrence T. Cherry, and James F. Carpenter—397(T)
Useful search coils and systems for uniform magnetic fields, Milan W. Garrett—136(T)
Vacuum spectrograph and its role in Lamb-Retherford shift, Ray Suchy—398(T)
Van de Graaff generator for demonstration purposes, Richard H. Waters—195(T)
Vapor pressure-temperature apparatus, Roy L. Judkins and G. P. Brewington—380
X-ray spectrometers, Kenneth E. Davis—483(T)

Cosmic rays
Cosmic-ray neutrons, Serge A. Korff—226
Near-neighbor effect in cosmic-ray stars, Kenneth M. King—483(T)
Origin of cosmic rays, Albrecht Unsöld—136(T)

Cosmography
Age determinations by radioactivity, H. R. Crane—136(T)
Origin of the lighter elements, William Band—251(A)
Structure and physical properties of interstellar gas clouds, Bengt Stromgren—443(T)

Courses
Advanced undergraduate course in radiation physics, laboratory experiments for, C. M. Zieman—399(T)
Block-and-gap physics course, flashback teaching technique applied to, A. J. Hatch and D. F. Cope—137
Chronology of modern physics, A. R. Tobey—167
Development of a course "Backgrounds of Science," Norris W. Goldsmith—330(A)
Electronics, laboratory course in, H. L. Schultz and W. G. Wadey—214
Engineering physics at Cornell, Lloyd P. Smith—174
Experimental course in reactor physics at the Oak Ridge School of Reactor Technology, E. C. Campbell—443(T)
Integrating course for the physics major, Carl E. Adams and Ralph A. Loring—250(T)
Modern physics, laboratory course in, Carl C. Sartain—443(A)
Preprofessional orientation course for physics majors, Stanley S. Ballard—330(A)
Special courses needed for nonscience majors, J. F. Mackell—399(T)
Specialized physics, C. Harrison Dwight—97
Teaching of physics to premedical students, J. K. Robertson—131(L)
Third-semester physics course, J. Gordon Stipe, Jr., and Isabel Boggs—443(A)
Unified approach to physics, Noel C. Little—351

Demonstrations
Apparatus for demonstrating osmosis, H. D. Smith—400(T)
Are x-ray tube demonstrations safe?, R. Schlegel and J. C. Lee—470
Automatic chart plotter for lecture room demonstrations, Alfred O. Nier and R. B. Thorness—416
Bernoulli's principle, demonstration of, E. Scott Barr—248(L)
BF_3 counter, demonstration of, Dale Marvin Holm—397(A)
Boyle's law—Dalton's law problem solved with gauge pressures. Centrifugal force with sound effects (demonstration), M. D. Adams—399(T)
Coriolis force, simple demonstration of, Arthur A. Klebba and Henry Stommel—247
Demonstrating harmonics and beats, Richard C. Hitchcock—329(A); 445
Demonstration experiments, Waldemar Noll—329(A)
Demonstrations with simple equipment, Gordon M. Dunning—482(A)
Diffraction and interference of sound, demonstration of, Everett K. Jenne—397(A)
Education?—or merely training?! VIII, a demonstration that resolved a dilemma, George Forster—195(A)
Electromagnetic induction, demonstration experiments in, D. S. Ainslie—232
Elliptic mirror for lecture demonstration, J. Smithson and W. T. Fenhagen—442(T)
Emission and absorption of sodium vapor, demonstration of, F. Blaha—130(L)
Experiment demonstration to determine rifle bullet velocity, Donald Worth—250(A)
Fermat's principle, model for demonstrating, W. Cullen Moore—1
High voltage and induction heating demonstrations, Richard Aurandt—398(T)
Hydrostatic paradox: phase II, demonstration of, Laurence E. Dodd—195(A)
Improved apparatus for demonstrating an oscillatory discharge, Edwin S. Fox—486(A)
Interference of light waves, demonstration of, Gordon M. Dunning—136(T)
Large-sized apparatus in lecture demonstrations in physics, W. H. Kadesch—483(T)
Lecture demonstration of coupled systems employing selsyn motors to provide variable coupling, Lloyd W. Morris—443(A)
Mass spectrometer, demonstration, F. E. Christensen—59
Methods of dynamic atomic energy demonstration, J. L. Kuranz—398(T)
Models for Rutherford and Thomson scattering, I. Walerstein—400(T)
Nuclear demonstration model, M. Olsen—398(T)
Nuclear ray tracks, demonstrations of, Cecil O. Riggs—482(A)

Phase difference between ordinary and extraordinary beams, demonstrations of, J. G. Winans—398(T)
Phase shift, demonstration of, Franklin Miller, Jr.—366
Physical model to demonstrate nuclear and paramagnetic resonance, E. F. Carr and C. Kikuchi—486(A)
Projection of small scale phenomena, Howard S. Seifert—195(A)
Relationships between ac and dc voltage, demonstration of, Albert V. Baez—399(A)
Research on natural illumination in school rooms, demonstration of, R. A. Boyd—136(T)
Resonant pendulums, demonstrations on, Ancil Thomas—196(T)
Rocket-propelled airplane, demonstration of, Vernon L. Bollman—195(A)
Science on television. A demonstration, Edward R. Bascom—485(T)
Selected simple demonstration experiments in mechanics, Richard W. Sutton—330(A)
Simple polarized light demonstration, Charles A. Fowler—398(A)
Surface tension, demonstration lecture on, Eric M. Rogers—328(T)
Synthetic rutile, Wilson W. Woodcock, Jr.—323(L)
Three lecture demonstrations, Francis W. Sears—329(A)
Van de Graaff generator for demonstration purposes, Richard H. Waters—195(T)
Water runs up hill, or does it?, John G. Betts—195(A)
Wave shaping demonstration, H. Fulton—196(T)
Yo-yo technics in teaching kinematics, Irving L. Kofsky—126

Department administration, maintenance, and activities (see Education, physics and science)
Engineering physics at Cornell, Lloyd P. Smith—174
Minimum curriculum for small departments, R. E. Martin—399(T)

Education, physics, and science
Engineering education, a bridge between ignorance and understanding, F. M. Dawson—486(T)
Enrollments and degrees awarded to physics majors, Marsh W. White—27
Graduate level laboratory training for government research scientists, George Abraham—487(A)
Place of original work in the undergraduate curriculum, D. E. Roller—399(T)
Putting the physics in the teaching of astrophysics, L. C. Green—442(A)
Should physics be assigned a fundamental role in the liberal arts curriculum?, Duane Roller—196(T)
Some reflections on the teaching of physics, John W. Hornbeck—328(T); 412
Some thoughts on teaching physics in technical schools, Paul F. Bartunek—483(T)
Starting point in physics teaching, Thornton Page—388(L)
Suggestions for the improvement of physics teaching in colleges and universities, George W. Hazzard—374
Teaching by publication, L. W. McKeehan—9
Teaching of physics, W. F. G. Swann—136(T); 182; 328(T)
Teaching of physics in Japan, M. S. Watanabe—400(T)
Television, physics via, Thomas P. Merritt—386(L)
Television, science on, Everett R. Phelps—387(L)
Training in undergraduate physics, C. H. Voelker—442(A)
Unified approach to physics, Noel C. Little—351
Who writes for the science teachers' magazines?, Robert S. Shaw—322(L); 331(A)

Electricity and magnetism
AC theory and practice, J. F. Mackell—400(T)
Analogous problems involving variable mass, moment of inertia, inductance, and capacitance, E. K. Chapin—6
Analysis of an *R–C* oscillator, Joseph H. Howey—85
Angular distribution of secondary electrons from platinum, John J. Faris—483(T)
B-H curves with a lecture table magnetometer, T. B. Brown—442(A)
Capacitance, measurement of, D. S. Ainslie—486(A)
Classical motion of a rigid charged body in a magnetic field, Herbert Goldstein—100
Deflection in cathode-ray tubes, J. W. Alinski—399(T)
Demonstration experiments in electromagnetic induction, D. S. Ainslie—232
Demonstration of phase shift using voltmeters, Franklin Miller, Jr.—366
Demonstration of the relationships between ac and dc voltage, Albert V. Baez—399(A)
Dynamic hysteresis loop tracer, T. A. Benham—136(T)
Education?—or merely training?! IX, Potential difference *vs* potential gradient, George Forster—398(A)
Electric and magnetic forces: A direct calculation, William Fuller Brown, Jr., I—290; II—333
Electric discharge in air at reduced pressure, Julius Sumner Miller—330(A)
Electrical analog computers, solution of differential equations by, Joseph L. Ryerson—90
Electromagnetic moment of inertia, W. R. Dixon—536
Electron theory of solids, John C. Slater—328(T); 368
Experiment for the direct measurement of magnetostatic fields, Edgar Everhart—474
Field emission at a billion amperes per cm², J. K. Trolan and W. P. Dyke—251(T)
Functional definitions and the meaning of electricity, F. W. Warburton—398(A)
High voltage and induction heating demonstrations, Richard Aurandt—398(T)
Inductance and capacitance, problems involving, E. K. Chapin—6
Laboratory course in electronics, H. L. Schultz and W. G. Wadey—214
Lines of force in electric and magnetic fields, Joseph Slepian—87
Magnetic recordings, R. J. Tinkham—196(T)
Magnetically maintained pendulum, Harold P. Knauss and Paul R. Zilsel—318
Modified Cotton balance, Zaboj V. Harvalik—128
Orientation of paramagnetic and diamagnetic rods in magnetic fields, Arthur R. Laufer—275
Output current of a nonlinear device, Bernard Salzberg—555
Polarization of electromagnetic waves, C. L. Andrews—159
Pool-type cathodes, Paul F. Copeland—249(T)
Potential against resistance—a graphical review, W. W. Sleator—262
Projects in electricity laboratory, Charles Williamson—482(A); 486(A)
Representation of the static polarization of rigid dielectrics by equivalent charge distributions, L. H. Fisher—73
Resistance network calculations. Physics and TV. An experiment with chalk. Cooling the professor, A. D. Hummel—399(T)
Sensitive arrangements of the Wheatstone bridge, Charles Williamson—123
Series impedance equivalent to two impedances in parallel, graphical solution for, Shou Chin Wang—178
Special case of self-inductance. A gadget or two, O. H. Smith—196(T)
Stroude and Oates induction bridge, W. H. Hyslop—483(T)
Survey of photoelectric theories, James J. Brady—397(A)
Uniform magnetic fields, useful search coils and systems for, Milan W. Garrett—136(T)
Use of the coulomb in electrostatic problems, Lester L. Skolil—245
Voltage measurements, R. C. Patton—399(T)

Employment of physicists
Education, employment, and earnings of physicists, Marsh White, M. H. Trytten, and Robert W. Cain—485(T)
Employment of physicists by the Department of the Army, Marsh W. White—257

Experiments
Advanced undergraduate laboratory experiments, manual of, T. B. Brown—145
Alternating current experiments at fifty cents each, Dean H. Harbour—399(A)
Anomaly in the determination of the coefficient of kinetic friction, Edward I. Rubendall—482(A)
Another attack on gamma (C_p/C_v), O. H. Smith—399(T)
B-H curves with a lecture table magnetometer, T. B. Brown—442(A)
Calibration of an ionization chamber for absolute x-ray beam power measurements, Duis D. Bolinger—397(T)
Criteria for choosing laboratory experiments, G. E. Owen—381
C. V. Boys' rainbow cup and experiments with thin films, John Satterly—448
Detection of soft x-rays with a scintillation counter, Lyman A. Webb, Ronald S. Paul, and Francis E. Dart—483(T)
Determination of γ for gases by self-sustained oscillations, laboratory experiment on, W. F. Koehler—113

Doubly refracting crystals, experiments with, Myron A. Jeppesen—81
Experiment demonstration to determine rifle bullet velocity, Donald Worth—250(A)
Experiment illustrating the elliptic integral of the first kind, George E. Owen and Daniel C. McKown—188
Experimental derivation of the Helmholtz dissonance curve using modern electrical apparatus, Newton Gaines—487(A)
High energy nucleon-nucleon scattering experiments at Berkeley, Geoffrey F. Chew and Burton J. Moyer—203
Humphrey Davy's heat-by-friction experiment, Duane Roller—196(T)
Improved mechanical equivalent of heat experiment, George G. Kretschmar—509
Investigation of gas amplification in a proportional counter, J. E. Hopson—250(A)
Knotty masspoint *versus* the rigid tinplate, Daniel A. Naymik—519
Laboratory exercise in nuclear emulsion technique, R. A. Peck, Jr., and Paul Stelson—48
Laboratory experiments for an advanced undergraduate course in radiation physics, C. M. Zieman—399(T)
Lauritsen quartz fiber electroscope, student experiments with, G. Karioris—398(T)
Magnetostatic fields, experiment for the direct measurement of, Edgar Everhart—474
Measurement of capacitance, motor-driven vibrator units for, D. S. Ainslie—486(A)
Measurement of thermal conductivity by Fitch's apparatus, John Satterly—132(L); Frank P. Fritchle—475(L)
Mechanics, selected simple experiments in, Richard M. Sutton—330(A)
Modern physics, undergraduate experiments in, W. C. Elmore—136(T)
Moment of inertia experiment, Willard L. Erickson—401
Novel uses of a ballistic galvanometer, Clayton M. Zieman—196(T)
Nuclear emulsions, experiments with, M. Elaine Toms—136(T)
On stirring a cup of tea, D. J. Montgomery—477(L)
Quantitative measurements of beta-radiation, R. S. Caswell—400(T)
Radial heat flow, laboratory experiment on, R. E. Sellers and E. Scott Barr—444(A)
Resultant and the meter stick, D. A. Naymik—400(T)
Self-sustained oscillations, experiment on, W. F. Koehler—113
Sensitive arrangements of the Wheatstone bridge, Charles Williamson—123
Simple method for locating principal points, Leonard Eisner—474
Snell's law, early experimental determination of, John W. Shirley—507
Spark timer and an impulse counter used as an inertia balance, H. Patterson—400(T)
Statistical fluctuations of radioactive decay, student experiment on, J. G. Grundle—398(T)
Testing the Rayleigh resolving power criterion, Frank Mooney—130(L)
Tinplate experiment for sophomore physics, D. A. Naymik—136(T)
Viscous fluid flow, experiments in, Karlem Riess and John E. Baudean—116
Which is the more accurate?, Harley J. Haden—189(L)

General education (see Education, physics and science)
Analogies as an aid to thought, Paul F. Bartunek—483(T)
On the analysis of transfer of training, Lewis A. Dexter and Robert A. Thornton—538
Harvard case histories in experimental studies, John W. Shirley—419

General physics, educational aspects
Elementary course, propagation of errors in, Richard Hanau—382(L)
Fluid dynamics in physics teaching, R. J. Seeger—486(T)
Fundamental problems of experimental physics, James Hough—489
Interesting aspects of teaching elementary fluid dynamics, R. M. Sutton—486(T)
Interesting type of problem in physics, R. R. Meijer—442(A)
Introduction of physics to freshmen, N. Goldowski—444(A)
Mathematical emphasis in undergraduate physics, John Phelps—122
Teaching atomic physics to engineers and other insolvable teaching problems, C. W. Ufford—442(A)
Teaching of physics to premedical students, J. K. Robertson—131(L)
Seminar for physics majors, W. C. Kelly—482(A)
Study tips for the physics student, James G. Potter—488(A)
Vector quantities in introductory physics, J. G. Winans—488(A)

General physics, instructional techniques
Atwood's machine and the teaching of Newton's second law, Irving L. Kofsky—354
Concerning classroom recitation, Julius Sumner Miller—476(L)
Effective recitations, O. H. Blackwood—482(T)
Efficiency in teaching measurements, William R. Varner—397(A)
Flashback teaching technique applied to a block-and-gap physics course, A. J. Hatch and D. F. Cope—137
Old-time classroom recitation: can it be restored?, Laurence Ellsworth Dodd—14
Photometric teaching methods using photoelectric cells, U. Andrewes and T. J. Dillon—514
Physics teaching in colleges and universities, suggestions for the improvement of, George W. Hazzard—374
Problems, Robert F. Clothier—321(L)
Teaching device, Julius Sumner Miller—130(L)
Teaching technics in kinematics, Irving L. Kofsky—126

Geophysics
Dangerous currents in the surf, Francis P. Shepard—195(T)
Heat and moisture balance of the earth's surface, study of, R. A. Bryson—398(T)
Oil industry, geophysics in, Charles P. Bazzoni—251(T)
Physics of the Chesapeake section, C. H. Voelker—442(A)
Radioactive ore in Boyle and some adjoining counties, Roy Ellis—250(T)
Rotating earth, motion relative to the surface of, Ralph Hoyt Bacon—52; Erratum—385(L)
Semidiurnal tidal oscillation of the earth's atmosphere, Harold L. Stolov—329(A); 403
Use of geophysics in groundwater investigations with applications to problems of import to Wisconsin, G. P. Woolard—398(T)

Heat and thermodynamics
Air temperature, sonic determination of, F. D. Watson and K. O. Lange—250(A)
Another attack on gamma (C_p/C_v), O. H. Smith—399(T)
Freezing in water pipes, Herbert M. Reese—425
Heat-sensitive color changes in some inorganic chemicals, C. H. Bachman and J. B. Maginnis—424
Heat transmission problems, Richard N. Lyon—486(T)
Improved mechanical equivalent of heat experiment, George G. Kretschmar—509
Laboratory experiment on radial heat flow, R. E. Sellers and E. Scott Barr—444(A)
Laboratory experiment on the determination of γ for gases by self-sustained oscillations, W. F. Koehler—113
Mathematics of elementary thermodynamics, John S. Thomsen—476(L); Karl Menger—476(L)
Maxwell's thermodynamic relations, John J. Gilvarry—131(L)
Measurement of thermal conductivity, Fitch's apparatus for, Frank P. Fritchle—475(L)
Regelation of ice, M. W. Zemansky—442(A)
Remarks on the use of Fitch's apparatus for the measurement of the thermal conductivity of thin slabs of poorly conducting materials, John Satterly—132(L)
Selective heating effect in the radiometer, Howard A. Carter—386(L)
Thermodynamics, Newton Gaines—484
Thermometers of the Royal Society, Louise Diehl Patterson—523
Use of curve differentials in thermodynamics, F. H. Crawford—284
Use of recording-controlling instruments in the intermediate heat laboratory, Joseph W. Straley—443(A)
Vapor pressure-temperature apparatus, Roy L. Judkins and G. P. Brewington—380
Well-informed heat engine, Richard C. Raymond—109

History and Biography
Chronology of modern physics, A. R. Tobey—167
Early experimental determination of Snell's law, John W. Shirley—507
Effect of the discovery of x-rays upon the scientific world as I remember it, W. P. Boynton—399(T)

Lectures of Professor Robert Pohl in Göttingen, Ernst Caspari—61
Lectures of the late Professor Benjamin W. Snow, John C. Blankenagel—60
Thermometers of the Royal Society, Louise Diehl Patterson—523

Industrial and governmental research (see Education, physics and science)
Geophysics in the oil industry, Charles P. Bazoni—251(T)
Government research scientists, graduate level laboratory training for, George Abraham—487(A)
Nuclear industry, metallurgical problems in, John H. Frye, Jr.—486(T)

Laboratory arts and techniques
Applications of ultrasonic pulse techniques in research and testing, Julian Frederick—486(T)
Growing piezoelectric crystals, Albert C. Walker—251(T)
Method for changing the response of a system, Edward P. Clancy—190(L)
Nondestructive testing of manufactured parts, G. P. Brewington—136(T)
Nuclear emulsion technique, laboratory exercise in, R. A. Peck, Jr., and Paul Stelson—48

Laboratory organization
Behavior objectives for laboratory instruction, Haym Kruglak—223
Criteria for choosing laboratory experiments, G. E. Owen—381
Departure in general physics laboratory procedure, Julius Sumner Miller—190(L)
Elementary laboratory for premedical students, Nora M. Mohler, Lilly Lorentz, and Elizabeth T. Bunce—170
General college physics, laboratory examination for, W. H. Kinsey and R. A. Rhodes, II—246
Laboratory guide for statics and dynamics, Earland Richie—250(T)
Laboratory in elementary physics, I. Walerstein—196(T)
Laboratory performance tests at the University of Minnesota, C. N. Wall, H. Kruglak, and L. E. H. Trainor—546
Laboratory tests, Benjamin H. Wender—438(L)
Modern physics, elementary laboratory in, Joseph W. Straley, Karl H. Fussler, and Paul E. Shearin—313
Practical laboratory experience, Richard D. Murphy—384(L)
Projects in electricity laboratory, Charles Williamson—482(A); 486(A)
Radiation physics, laboratory experiments for, C. M. Zieman—399(T)
Student questionnaires as an aid to laboratory teaching techniques, Willard Geer—564

Language
Causality, relativity, and language, Elihu Fein—211; 439(L); Raymond J. Munick—438(L)

Light (see Apparatus)
Basic principle for the telescope and microscope, H. C. Schepler and A. N. Smith—129
Colorimetry and its applications, H. M. Sullivan—483(T)
Conservation of tangential momentum, Snell's law equivalent to, Frank Mooney—385(L)
Correction to the treatment of Fresnel diffraction, C. L. Andrews—280
Demonstrating Fermat's principle, model for, W. Cullen Moore—1
Dispersion and resolving power of prism spectrometers, George W. Hazzard—235
Early experimental determination of Snell's law, John W. Shirley—507
Emission and absorption of sodium vapor, demonstration of, F. Blaha—130(L)
Experiments with doubly refracting crystals, Myron A. Jeppesen—81
Forms of Cartesian ovals in an optical range, H. W. Farwell—454
Fraunhofer multiple-slit diffraction patterns with infinite sources, A. E. Smith and C. D. Hause—488(A)
Further note on measurement of wavelength with a diffraction grating Louis R. Weber and Don L. Hammond—562
Illumination in school rooms, demonstration of research on, R. A. Boyd—136(T)
Infrared spectrometry and the reflecting microscope, Robert C. Gore—251(T)
Interference of light waves, demonstration of, Gordon M. Dunning—136(T)
Lenses for laboratory-built spectrographs, Ralph A. Loring—487(A)
Lenses for spectrographs, Ralph A. Loring—400(T)
Mercury light source, Wallace A. Hilton—248(L)
Optical quarter-wave and half-wave plates, microwave analog of, G. Birnbaum—442(A)
Penetration into the rarer medium in total reflection, Maynard Dean Pearson—397(T)
Phase difference between ordinary and extraordinary beams, demonstrations of, J. G. Winans—398(T)
Photometric teaching methods using photoelectric cells, U. Andrewes and T. J. Dillon—514
Polarization of electromagnetic waves, C. L. Andrews—159
Proposed international standards of sign conventions and symbols for geometrical optics, Stanley S. Ballard—122; 328(A)
Rayleigh resolving power criterion, testing of, Frank Mooney—130(L)
Recent work in infrared spectroscopy at Northwestern University, Russel A. Fisher—249(T)
Seeing light and color, Lyle Brewer—328(A)
Simple method for locating principal points, Leonard Eisner—474
Simple polarized light demonstration, Charles A. Fowler—398(A)
Simple relation for an achromatic telescope objective, D. G. Dhavale—379
Studies of transmission zone plates, Ora E. Myers, Jr.—359
Survey of photoelectric theories, James J. Brady—397(A)
Synthetic rutile, Wilson W. Woodcock, Jr.—323(L)
Thin lenses, theorem on deviation in, W. T. Payne—57
Tracing of skew rays by analytical methods, Charles C. Dalton—250(A)

Mathematics
Binary numeration before Leibniz, John W. Shirley—452
Curve differentials in thermodynamics, use of, F. H. Crawford—284
Elementary thermodynamics, mathematics of, John S. Thomsen—476(L); Karl Menger—476(L)
Elliptic integral of the first kind, experiment illustrating, George E. Owen and Daniel C. McKown—188
Forms of Cartesian ovals in an optical range, H. W. Farwell—454
Graphical solution for the series impedance equivalent to two impedances in parallel, Shou Chin Wang—178
Implication of the Laplace transformation, Horace M. Trent—437(L)
Order parameters, Martin J. Klein—153
Propagation of errors in the elementary course, Richard Hanau—382(L)
Simple geometrical proof of Buckingham's π-theorem, Stanley Corrsin—180
Solution of differential equations by electrical analog computers, Joseph L. Ryerson—90
Solution of the Schroedinger equation for an approximate atomic field, E. H. Kerner—136(T)
Standard deviation, physical values which depend on, J. J. Bikerman—58
Theorem on deviation in thin lenses, W. T. Payne—57
Tracing of skew rays by analytical methods, Charles C. Dalton—250(A)
Undergraduate physics, mathematical emphasis in, John Phelps—122

Mechanics (see Apparatus)
About an argument of Newton, Zahur Hussain—197
Application of dimensional analysis, S. F. Borg—69
Application of the generalized area-moment propositions, A. W. Simon—35
Body falling in a resisting medium of variable density, W. A. Bowers—562
Center of percussion, W. W. Sleator—486(A)
Circular translation, W. W. Sleator—136(T)
Compressible flows, James B. Kelley—237
Demonstration of Bernoulli's principle, E. Scott Barr—248(L)
Does pressure have direction?, R. R. Dempster—64(L)
Driving of tent stakes, John S. Rinehart—562
Effect of a constant force on a particle in a box, Vernon Myers—329(A)

Elementary fluid dynamics, interesting aspects of, R. M. Sutton—486(T)
Energy density in a gravitational field, John A. Eldridge—63(L)
Energy independent of mass in simple harmonic motion, Laurence E. Dodd—189(L)
Experiments in viscous fluid flow, Karlem Riess and John E. Baudean—116
Extended Bernoulli equation, A. Klinkenberg and G. J. Sleutelberg—435(L)
Extension of the falling chain problem, Julius Sumner Miller—383(L)
Falling chains, John Satterly—383(L)
Fluid dynamics in physics teaching, R. J. Seeger—486(T)
Has pressure direction?, Michael Danos—248(L)
Kinematics, teaching technics, Irving L. Kofsky—126
Knotty masspoint *versus* the rigid tinplate, Daniel A. Naymik—519
Modes of vibration of a rotating string, John S. O'Connor—136(T)
Moment of inertia, problems involving, E. K. Chapin—6
Motion of a particle through a resisting medium of variable density, Ralph Hoyt Bacon—64(L); William Squire—426
Motion relative to the surface of the rotating earth, Ralph Hoyt Bacon—52; Erratum—385(L)
Newtonian mechanics and the equivalence of gravitational and inertial mass, Leonard T. Pockman—305
Newton's law of attractions, Zahur Hussain—146
Pressure energy and Bernoulli's principle, G. A. Lindsay—400(T); 487(A)
Problems involving variable mass, John S. Thomsen—435(L); Ernest K. Chapin—436(L)
Rocking of an elliptic band, John Satterly—511
Selected simple demonstration experiments in mechanics, Richard M. Sutton—330(A)
Semidiurnal tidal oscillation of the earth's atmosphere, Harold L. Stolov—329(A); 403
Simple demonstration of Coriolis force, Arthur A. Klebba and Henry Stommel—247
Simple dynamics and the value of g, John Satterly—322(L)
Snell's law equivalent to the conservation of tangential momentum, Frank Mooney—385(L)
Statics and dynamics, laboratory guide for, Earland Richie—250(T)
Study of centrifugal force and rotational inertia, apparatus for, W. L. Kennon—443(A)
Variable mass, problems involving, E. K. Chapin—6
Weight of a falling chain, Julius Sumner Miller—63(L)

Mesons

Detection of the neutral meson, Lawrence S. Germain—251(T)
Properties of mesons and nucleons today, Philip Morrison—251(T)

Microwaves

Microwave analog of the optical quarter-wave and half-wave plates, G. Birnbaum—442(A)
Microwave lenses, focusing sound waves with, Winston E. Kock—328(T)
Microwave reflection from water spheres, A. L. Aden—163
Polarization of electromagnetic waves, C. L. Andrews—159

Modern physics

Chronology of modern physics, A. R. Tobey—167
Elementary laboratory in modern physics, Joseph W. Straley, Karl H. Fussler, and Paul E. Shearin—313
Laboratory course in modern physics, Carl C. Sartain—443(A)
Place of recent modern physics in the curriculum, A. D. Sprague—399(T)
Undergraduate experiments in modern physics, W. C. Elmore—136(T)

Nuclear physics

Alpha-radioactivity, illustrating the regularity of, F. W. Van Name, Jr.—230
Capture of orbital electrons by nuclei (K-capture), C. Sharp Cook—37
Collisions of high energy nuclear particles with nuclei, B. J. Moyer and G. F. Chew—17
Demonstrations of nuclear ray tracks, Cecil O. Riggs—482(A)
Depolarization in scattering of thermal neutrons, David B. Nicodemus—483(T)
Design and construction of a solid boron neutron detector, Dale W. Holm—483(T)
Energy loss of beta-rays in solid anthracene, Ronald S. Paul—483(T)
Geiger-Nuttall law and regularity in alpha-radioactivity, F. W. Van Name—136(T)
High energy nucleon-nucleon scattering experiments at Berkeley, Geoffrey F. Chew and Burton J. Moyer—203
Laboratory exercise in nuclear emulsion technique, R. A. Peck, Jr., and Paul Stelson—48
Metallurgical problems in nuclear industry, John H. Frye, Jr.—486(T)
Molecular beam researches in nuclear and electronic physics, I. I. Rabi—328(T)
Neutron detection methods and the demonstration of a BF_3 counter, Dale Marvin Holm—397(A)
Neutron refractive indices, C. Kikuchi—400(T)
Neutrons as waves and particles, C. G. Shull—443(T)
Nuclear demonstration model, M. Olsen—398(T)
On illustrating the regularity of alpha-radioactivity, F. W. Van Name, Jr.—230
Radioactive disintegration, Robert Katz—389(L)
Some chemical research problems in the development of nuclear reactors, J. A. Swartout—486(T)
Student experiment on the statistical fluctuations of radioactive decay using a scaler, J. G. Grundle—398(T)
Systematics of alpha-decay, E. G. Ebbighausen—483(T)

Philosophy of science

Causality, relativity, and language, Elihu Fein—211; 439(L); 566(L); Raymond J. Munick—438(L)
Relative causality?, Lee Spetner—565(L)

Properties of matter

Coulomb friction in the motion of two blocks, J. S. Thomsen—442(T)
Effect of aeration on the viscosity of water, Charles H. Tindal and John B. Mason—382
Electron theory of solids, J. C. Slater—328(T); 368
Friction phenomena, comments on, G. P. Brewington—357
Heat-sensitive color changes in some inorganic chemicals, C. H. Bachman and J. B. Maginnis—424
Osmosis, apparatus for demonstrating, H. D. Smith—400(T)
Static polarization, representation of, L. H. Fisher—73
Study of sliding friction, Charles A. Maney—487(A)
Surface tension, demonstration lecture on, Eric M. Rogers—328(T); 443(T)
Synthesis of atoms and their isotopes, Robert C. Colwell—481(A)
Thermal conductivity, Fitch's apparatus for the measurement of, John Satterly—132(L); Frank P. Fritchle—475(L)

Radio and television

Multiple frequency standard employing a modulated television-type raster for comparison of frequencies, Thomas J. Yeadon and Lloyd W. Morris—444(A)
Physics and TV, A. D. Hummel—399(T)
Physics via television, Thomas P. Merritt—386(L)
Science on television, Everett R. Phelps—387(L)
Science on television, a demonstration, Edward R. Bascom—485(T)
Sound experiments suitable for television, Edward Bascom—400(T)
Television—demonstration, J. W. Alinsky—196(T)
WFIL-TV University of the air, Armand L. Hunter—485(T)

Reports, announcements, and news

Books received—135
Colloquium of college physicists—256
Engineering physics at Cornell, Lloyd P. Smith—174
Fellowships for women, 1952–53—356
Fulbright awards for 1952–53—350
Manual of advanced undergraduate laboratory experiments, T. B. Brown—145
Meeting of the Pennsylvania Conference of College Physics Teachers—136
Meetings of the Michigan Teachers of College Physics—136; B. H. Dickinson—400
National balance sheet in June 1951, E. A. Walker—485(T)
National science foundation developments, A. A. Potter—486(T)
National science foundation fellowships—535
Nation's potential in educational institutions, A. F. Spilhaus—486(T)
Ninth Annual Pittsburgh Diffraction Conference—378
Physics Club of Philadelphia, Mabel A. Purdy—251

Physics in the news today, Pearl I. Young—136(T)
Proposal of the International Commission of Optics for international standardization of sign conventions and symbols in geometrical optics, Stanley S. Ballard—122
Recent work in infrared spectroscopy at Northwestern University, Russel A. Fisher—249(T)
Report of the East Lansing meeting of AAPT, C. D. Hause—484
Report on the New York Meeting, Raymond T. Ellickson—397(T)
Solomon's House, T. A. Boyd—484(T)
Story of Palomar, Ira S. Bowen—196(T)
Symposium on molecular structure and spectroscopy—115
University of Alabama Physics Building Dedication meeting, Eric Rodgers—255
Work of the Canadian atomic energy project, D. A. Keys—136(T)
World trends in the publication of physical research, 1938–1948, John J. McCarthy—79

Rockets
Demonstration of a rocket-propelled airplane, Vernon L. Bollman—195(A)
Rockets and jet propulsion, Charles A. Boyd—398(T)

Secondary-school physics
Preparation of high school teachers and certificating by the state—196(T)
Spiral program in high school physics, Alexander Efron—331(A)

Social and economic aspects of science (see Education, physics and science)
Physics and the emergency, F. G. Slack—443(T)
Physics in the civil defense program, G. E. C. Kauffman—442(A)
Scientists and mobilization, Science Advisory Committee—473(A)

Sound (see Apparatus)
Acoustic interferometer, K. J. Metzgar—482(A)
Chladni plate figures, W. M. Pierce—436(L)
Concerning some primitive musical instruments, Julius Sumner Miller—565(L)
Constructive and destructive interference, Grant O. Gale—321
Demonstrating harmonics and beats, Richard C. Hitchcock—329(A); 445
Dropping a stone down a shaft, Julius Sumner Miller—436(L)
Experimental derivation of the Helmholtz dissonance curve using modern electrical apparatus, Newton Gaines—487(A)
Focusing sound waves with microwave lenses, Winston E. Kock—328(T)
Lissajous figures in Melde's experiment, Julius Sumner Miller—249(L)
Longitudinal waves, W. W. Sleator—487(A)
Magnetic recordings, R. J. Tinkham—196(T)
Methods of making ultrasonic fields visible, Egon A. Hiedemann—486(T)
Physical basis of piano tuning, Ora L. Railsback—249(T)
Power measurements in ultrasonics, Oskar Mattiat—486(T)
Replies to inquiring letters, John Satterly—191(L)
Some problems in the recording of classical music, Harry L. Robin—328(T)
Sonic determination of air temperature, F. D. Watson and K. O. Lange—250(A)
Television, sound experiments suitable for, Edward Bascom—400(T)
Transient characteristics of the tuba, Ervin Holland-Moritz—400(T)
Ultrasonic effects—film—486(T)
Ultrasonic pulse techniques, applications of, Julian Frederick—486(T)
Use of a sonic anemometer in micrometeorology and the study of the heat and moisture balance at the earth's surface, R. A. Bryson—398(T)
Wave fronts, attenuation, and diffraction, W. Cullen Moore—331(A)

Teacher training (see Education, physics and science)
Graduate training for college physics teachers, J. F. Mackell—196(T)
Preparation of high school teachers and certificating by the state—196(T)
Panel discussion of committee on improvement of teaching, L. E. Grinter, H. P. Hammond, A. P. Colburn, and J. F. Calvert—486(T)
Practical laboratory experience, Richard D. Murphy—384(L)
Report on requirements for physics teachers, M. E. Hufford—196(T)
Teacher training for the technical institute, J. B. Hershman—196(T)

Testing, theory and techniques
Bonus questions and recommended reading, Roald K. Wangsness—475(L)
Laboratory examination for general college physics, W. H. Kinsey and R. A. Rhodes, II—246
Laboratory performance tests at the University of Minnesota, C. N. Wall, H. Kruglak, and L. E. H. Trainor—546
Laboratory tests, Benjamin H. Wender—438(L)
Reduction of test grades to standard distribution, H. Bowldon—400(T)
What constitutes a laboratory examination?, Julius Sumner Miller—191(L)

Textbooks
Book Review: *Advances in radiochemistry* by E. Broda, Max T. Rogers—480
Book Review: *Adventure into the unknown* by Lawrence A. Hawkins, L. B. Ham—430
Book Review: *Albert Einstein—philosopher-scientist* by Paul Arthur Schilpp, Herbert Jehle—252
Book Review: *Applied nuclear physics* by Ernest Pollard and William L. Davidson, Sanborn C. Brown—429
Book Review: *Atomic physics* by Wolfgang Finkelnburg, Ralph B. Bowersox—134
Book Review: *Chemistry visualized and applied* by A. J. Courchaine, Frederic B. Dutton—393
Book Review: *Crystal growth* by H. E. Buckley, A. C. Walker—430
Book Review: *Economic aspects of atomic power* by Sam H. Schurr and Jacob Marschak, Jacob Schmookler—480
Book Review: *Electricity and magnetism* by Norman E. Gilbert, G. K. Schoepfle—193
Book Review: *Electromagnetic waves and radiating systems* by Edward C. Jordan, C. L. Andrews—477
Book Review: *Engineering thermodynamics* by Herman J. Stoever, C. H. Pesterfield—567
Book Review: *Experimental spectroscopy* by Ralph A. Sawyer, Forrest F. Cleveland—567
Book Review: *First principles of atomic physics* by R. E. Humphreys and R. Beringer, R. L. Edwards—67
Book Review: *From the life of a researcher* by William Weber Coblentz, G. W. Stewart—427
Book Review: *Fundamentals of acoustics* by Lawrence E. Kinsler and Austin R. Frey, G. S. Bennett—254
Book Review: *Fundamentals of quantum mechanics* by Enrico Persico translated and edited by Georges M. Temmer, Melvin Lax—478
Book Review: *Heat and temperature measurement* by Robert L. Weber, W. F. Koehler—194
Book Review: *Hydrodynamics* by Garrett Birkhoff, Milton S. Plesset—479
Book Review: *Introduction to electricity and optics* by Nathaniel H. Frank, H. A. Nye—194
Book Review: *Introduction to the study of physics* by Wolfgang Finkelnburg, H. Margenau—433
Book Review: *Introductory nuclear physics* by David Halliday, G. E. Page—252
Book Review: *Laplace transformation* by William T. Thomson, Horace M. Trent—391
Book Review: *Methods and materials for teaching general and physical science* by John S. Richardson and G. P. Cahoon, Victor H. Noll—478
Book Review: *Methods in climatology* by V. Conrad and L. W. Pollak, John G. Albright—65
Book Review: *Microwave electronics* by John C. Slater, Sherwood K. Haynes—133
Book Review: *Molecular spectra and molecular structure* by Gerhard Herzberg, S. Mrozowski—390
Book Review: *Photons and electrons* by K. H. Spring, Gerhart Groetzinger—192
Book Review: *Physics* by S. G. Starling and A. J. Woodall, John Satterly—393
Book Review: *Physics, its laws, ideas, and methods* by Alexander Kolin, W. H. Michener—392
Book Review: *Quantum mechanics of particles and wave fields* by Arthur March, James H. Bartlett—568

Book Review: *Reflections of a physicist* by P. W. Bridgman, Oswald Blackwood—66
Book Review: *Relativity physics* by W. H. McCrea, H. C. Corben—132
Book Review: *Response of physical systems* by John Dezendorf Trimmer, Ralph Hoyt Bacon—133
Book Review: *Technical optics, Volume II* by L. C. Martin, Donald H. Jacobs—326
Book Review: *The autobiography of Robert A. Millikan*, B. J. Spence—65
Book Review: *The climate near the ground* by Rudolf Geiger, George J. Bouyoucos—192
Book Review: *The friction and lubrication of solids* by F. P. Bowden and D. Tabor, Frederic Palmer—428
Book Review: *The heavens above* by J. B. Sidgwick, Harry M. Bendler—429
Book Review: *The nature of physical reality* by Henry Margenau, R. B. Lindsay—67
Book Review: *The new physics* by Sir C. V. Raman, Thomas P. Merritt—434
Book Review: *The origin of the earth* by W. M. Smart, Thornton Page—432
Book Review: *The principles of cloud-chamber technique* by J. G. Wilson, Wayne E. Hazen—427
Book Review: *The special theory of relativity* by Herbert Dingle, H. C. Corben—132
Book Review: *Thermodynamics* by Francis Weston Sears, Richard C. Raymond—193
Book Review: *Ultrasonics* by P. Vigoureux, Robert Lagemann—567
Book Review: *Vacuum equipment and techniques* by A. Guthrie and R. K. Wakerling, Thomas H. Osgood—431
Book Review: *Wave guides* by H. R. L. Lamont, R. D. Spence—326
Electrostatic problems, use of the coulomb in, Lester L. Skolil—245
Harvard case histories in experimental science; the evolution of an idea, John W. Shirley—419
Laboratory manual to be produced as a memorial to Lloyd William Taylor, T. B. Brown—323(L)
Lumen, Harry Peach—322(L)
Taylor memorial laboratory manual, Joseph D. E'der—537

Units, dimensions, and terminology
Application of dimensional analysis, S. F. Borg—69
Designation of a thyratron tube, Carl C. Sartain—389(L)
Lumen, Harry Peach—322(L)
Matter of terminology: the kilocalorie and the kilomole, Francis E. Throw—436(L)
Order parameters, Martin J. Klein—153
Physical values which depend on the standard deviation of another value, J. J. Bikerman—58
Sign conventions and symbols for geometrical optics, proposed international standards of, Stanley S. Ballard—122; 328(A)
Slugging out a case for the pounders, J. G. Winans—439(L); a reply, S. L. Gerhard—440(L)
Use of the coulomb in electrostatic problems, Lester L. Skolil—245

Visual materials and methods
Illustrating the regularity of alpha-radioactivity, F. W. Van Name, Jr.—230
Methods of making ultrasonic fields visible, Egon A. Hiedemann—486(T)
Potential against resistance—a graphical review, W. W. Sleator—262
Projection of small scale phenomena, Howard S. Seifert—195(A)
Teaching physics, study of subjects for motion pictures for, Robert Petry—487(A)
Ultrasonic effects—film—486(T)

X-rays
Are x-ray tube demonstrations safe?, R. Schlegel and J. C. Lee—470
Beam power measurements by an ionization chamber, Duis D. Bollinger—397(T)
Detection of soft x-rays with a scintillation counter, Lyman A. Webb, Ronald S. Paul, and Francis E. Dart—483(T)
One- and two-dimensional x-ray diffraction, B. D. Cullity—500
Radiation intensities near a demonstration x-ray tube, R. Schlegel—400(T)
X-rays, effect of discovery of, W. P. Boynton—399(T)
X-ray powder patterns, camera for, L. F. Connell, Jr., and H. C. Martin, Jr.—127
X-ray spectrometers, Kenneth E. Davis—483(T)

The mathematical form in which new theories are usually dressed is the method by which we assure ourselves that our reasoning is logically sound, and therefore that it can be applied in unfamiliar situations. In his admirable work, "The Nature of the Physical World." Eddington has said that whereas in the nineteenth century the Creator was regarded as an engineer, in the twentieth he is a pure mathematician. I do not agree with this theological diagnosis. Every new *body of discovery is mathematical in form, because there is no other guidance that we can have. But as familiarity grows we find unsuspected analogies with our previous experience, and the engineer gets his chance of replacing the mathematician. When Eddington spoke of the nineteenth century he was thinking of the middle and end of it; if he had looked at the beginning he would have seen that the great discoveries of optical theory began life in mathematical form too.*—C. G. DARWIN, THE NEW CONCEPTIONS OF MATTER, 1931.